

Lecture 5  
2018/2019

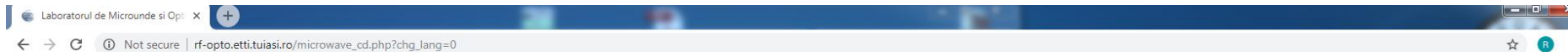
# Microwave Devices and Circuits for Radiocommunications

# 2018/2019

- 2C/1L, **MDCR**
- Attendance at minimum 7 sessions (course + laboratory)
- Lectures- **associate professor Radu Damian**
  - Friday 09-11, 11.13
  - E – 50% final grade
  - problems + (2p atten. lect.) + (3 tests) + (bonus activity)
    - 3p=+0.5p
  - all materials/equipments authorized
- Laboratory – **associate professor Radu Damian**
  - Wednesday 12-14, 11.12 odd weeks
  - L – 25% final grade
  - P – 25% final grade

# Materials

■ <http://rf-opto.etti.tuiasi.ro>



Main **Courses** Master Staff Research Students Admin

**Microwave CD** Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

## Microwave Devices and Circuits for Radiocommunications (English)

### Course: MDCR (2017-2018)

**Course Coordinator:** Assoc.P. Dr. Radu-Florin Damian  
**Code:** EDOS412T  
**Discipline Type:** DOS; Alternative, Specialty  
**Credits:** 4  
**Enrollment Year:** 4, Sem. 7

### Activities

**Course:** Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:  
**Laboratory:** Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

### Evaluation

Type: **Examen**

**A:** 50%, (Test/Colloquium)  
**B:** 25%, (Seminary/Laboratory/Project Activity)  
**D:** 25%, (Homework/Specialty papers)

### Grades

[Aggregate Results](#)

### Attendance

[Course](#)  
[Laboratory](#)

### Lists

[Bonus-uri acumulate \(final\)](#)  
[Studenti care nu pot intra in examen](#)

### Materials

#### Course Slides

[MDCR Lecture 1](#) (pdf, 5.43 MB, en, [ps](#))  
[MDCR Lecture 2](#) (pdf, 3.67 MB, en, [ps](#))  
[MDCR Lecture 3](#) (pdf, 4.76 MB, en, [ps](#))  
[MDCR Lecture 4](#) (pdf, 5.58 MB, en, [ps](#))

# Photos

Nr. Student	Student	Prezent	Nr. Student	Student	Prezent	Nr. Student	Student	Prezent
1	ANGHELUS IONUT-MARIUS	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	2	ANTIGHIN FLORIN-RAZVAN	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	3	ANTONICA BIANCA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:
4	APOSTOL PAVEL-MANUEL	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	5	BALASCA VALIAN-PETRU	<input checked="" type="checkbox"/> Puncte: 0 Nota: 0 Obs:	6	BOSTAN ANDREI-PETRICIA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:
7	BOTEZAT EMANUEL	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	8	BUTUNOI GEORGE-MADALIN	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	9	CHILEA SALUCA-MARIA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:
10	CHERITOIU ECATERINA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:	11	COJOC MARIUS	<input checked="" type="checkbox"/> Puncte: 0 Nota: 0 Obs:	12	COJOCARIU AURA-FLORINA	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:

Nr. Student	Student	Prezent
2	ANTIGHIN FLORIN-RAZVAN	<input type="checkbox"/> Puncte: 0 Nota: 0 Obs:

# Access

## ■ Not customized



A student profile page with a photo of a man on the left. To the right, under the heading "Date:", is a table with student details. Below this is a link "Acceseaza ca acest student" circled in red. At the bottom is a table titled "Note obtinute" showing grades for "Tehnologii Web".

**Date:**

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

[Acceseaza ca acest student](#)

**Note obtinute**

Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW			Tehnologii Web			
N		17/01/2014	Nota finala	10	-	
A		17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
B		17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
D		17/01/2014	Tema Tehnologii Web 2013/2014	9	-	



A login form with a light blue background. It contains fields for "Nume" (with "IACOBSCUIN" entered and redacted), "Email", and "Cod de verificare" (with "344bd9f" entered and redacted). A "Trimite" button is at the bottom.

Nume  
IACOBSCUIN

Email

Cod de verificare  
344bd9f

Trimite

# Examen: Logarithmic scales

$$\text{dB} = 10 \cdot \log_{10} (P_2 / P_1)$$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

$$\text{dBm} = 10 \cdot \log_{10} (P / 1 \text{ mW})$$

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 $\mu$ W
-30 dBm	= 1 $\mu$ W
-60 dBm	= 1 nW

$$[\text{dBm}] + [\text{dB}] = [\text{dBm}]$$

$$[\text{dBm/Hz}] + [\text{dB}] = [\text{dBm/Hz}]$$

$$[\text{x}] + [\text{dB}] = [\text{x}]$$

# Examen

- Complex numbers arithmetic!!!!
- $z = a + j \cdot b ; j^2 = -1$

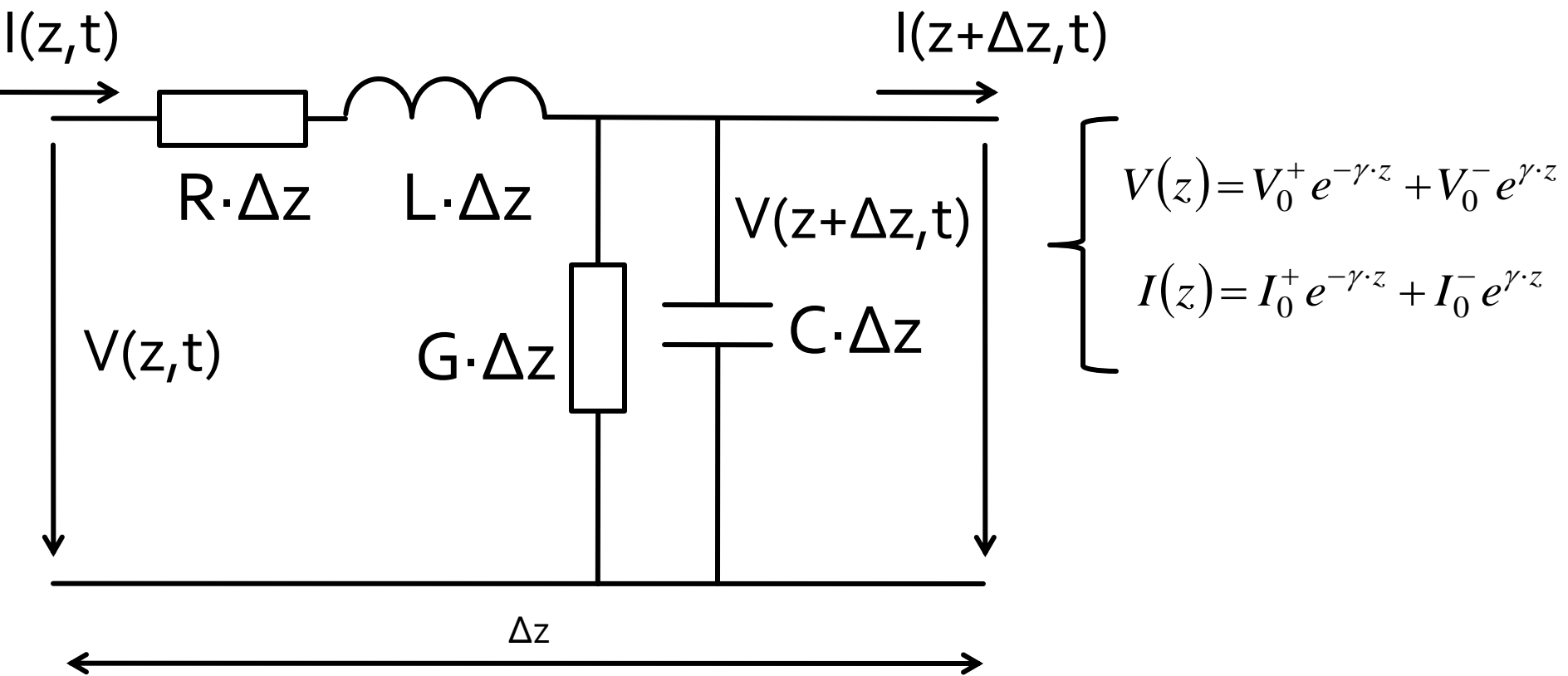
# TEM transmission lines

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# Transmission line equivalent model

- TEM wave propagation, at least two conductors



# The lossless line

$$V(z) = V_0^+ \cdot (e^{-j\beta \cdot z} + \Gamma \cdot e^{j\beta \cdot z}) \quad I(z) = \frac{V_0^+}{Z_0} \cdot (e^{-j\beta \cdot z} - \Gamma \cdot e^{j\beta \cdot z})$$

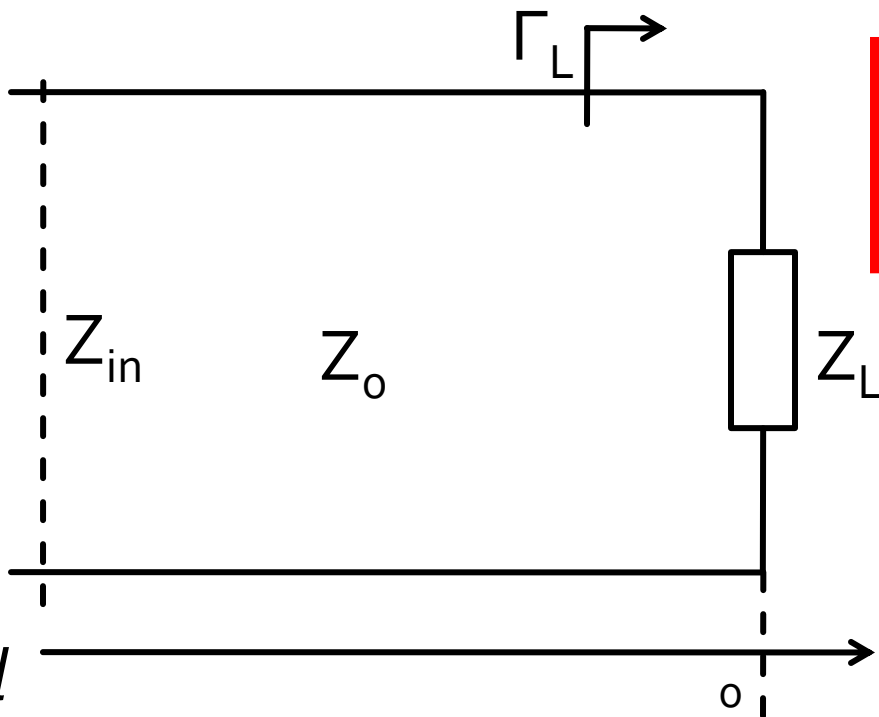
- time-average Power flow along the line

$$P_{avg} = \frac{1}{2} \cdot \text{Re}\{V(z) \cdot I(z)^*\} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \text{Re}\left\{1 - \Gamma^* \cdot \underbrace{e^{-2j\beta \cdot z} + \Gamma \cdot e^{2j\beta \cdot z}}_{(z - z^*) = \text{Im}} - |\Gamma|^2\right\}$$
$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot (1 - |\Gamma|^2)$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB]  $RL = -20 \cdot \log|\Gamma|$  [dB]

# The lossless line

- input impedance of a length  $l$  of transmission line with characteristic impedance  $Z_0$ , loaded with an arbitrary impedance  $Z_L$



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

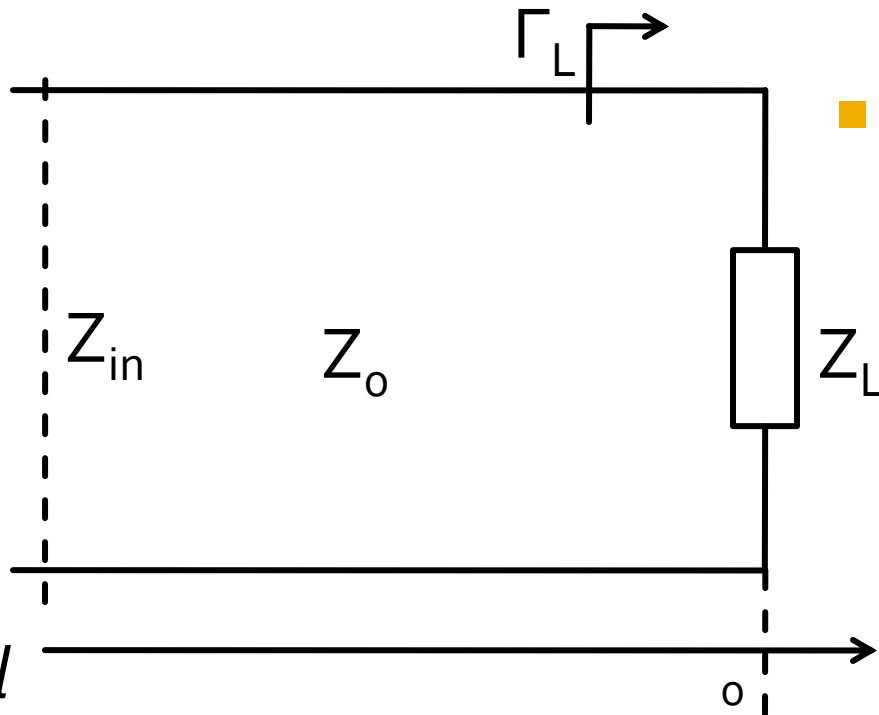
# The lossless line, special cases

- $l = k \cdot \lambda/2$       $\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi$       $\tan \beta \cdot l = 0$

$$Z_{in} = Z_0$$

- $l = \lambda/4 + k \cdot \lambda/2$       $\tan \beta \cdot l \rightarrow \infty$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$



- quarter-wave transformer

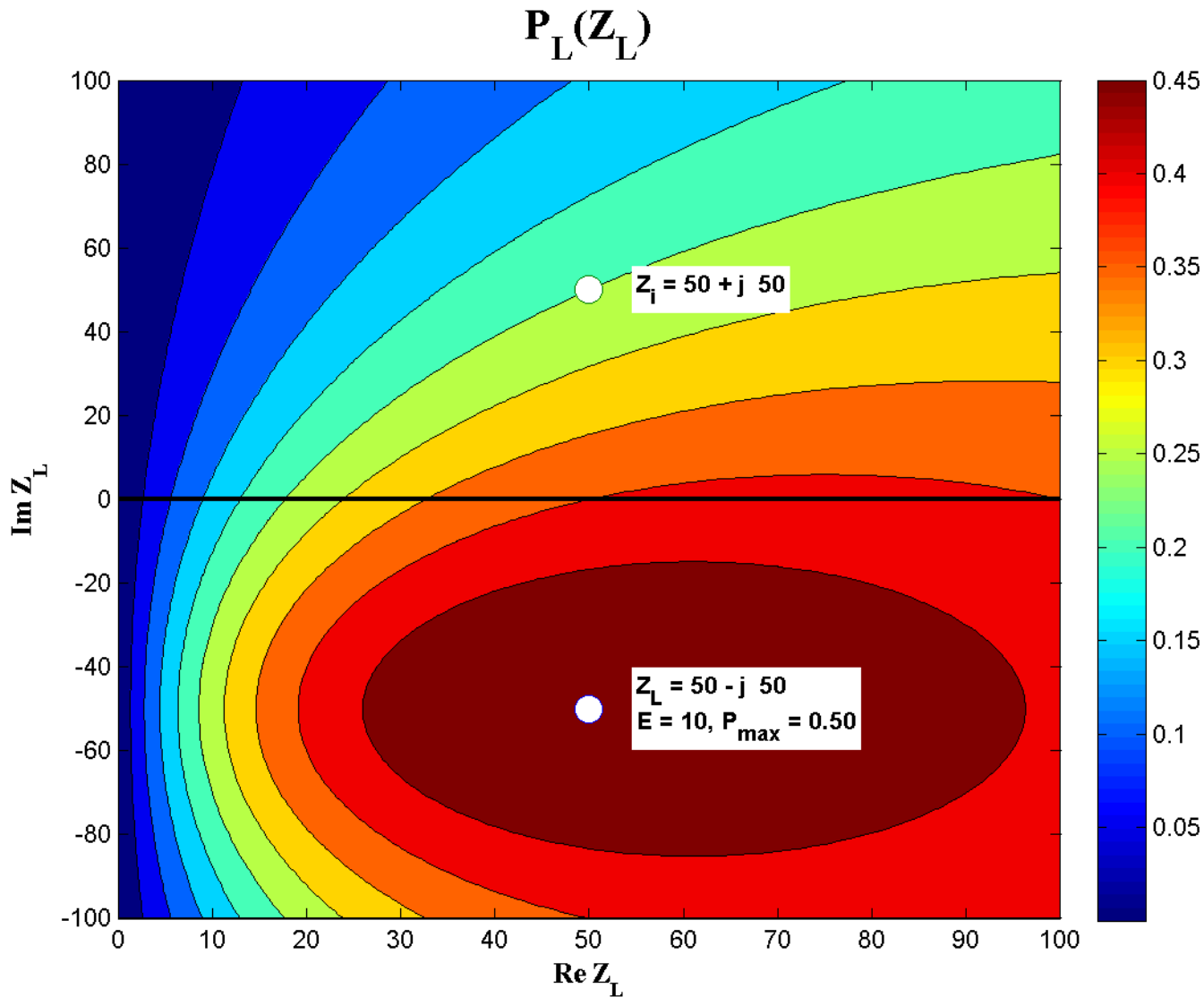
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Impedance Matching with Impedance Transformers (Lab 1)

# Impedance Matching

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# Matching, complex impedances



$$Z_L = Z_i^*$$

# Matching , from the point of view of power transmission

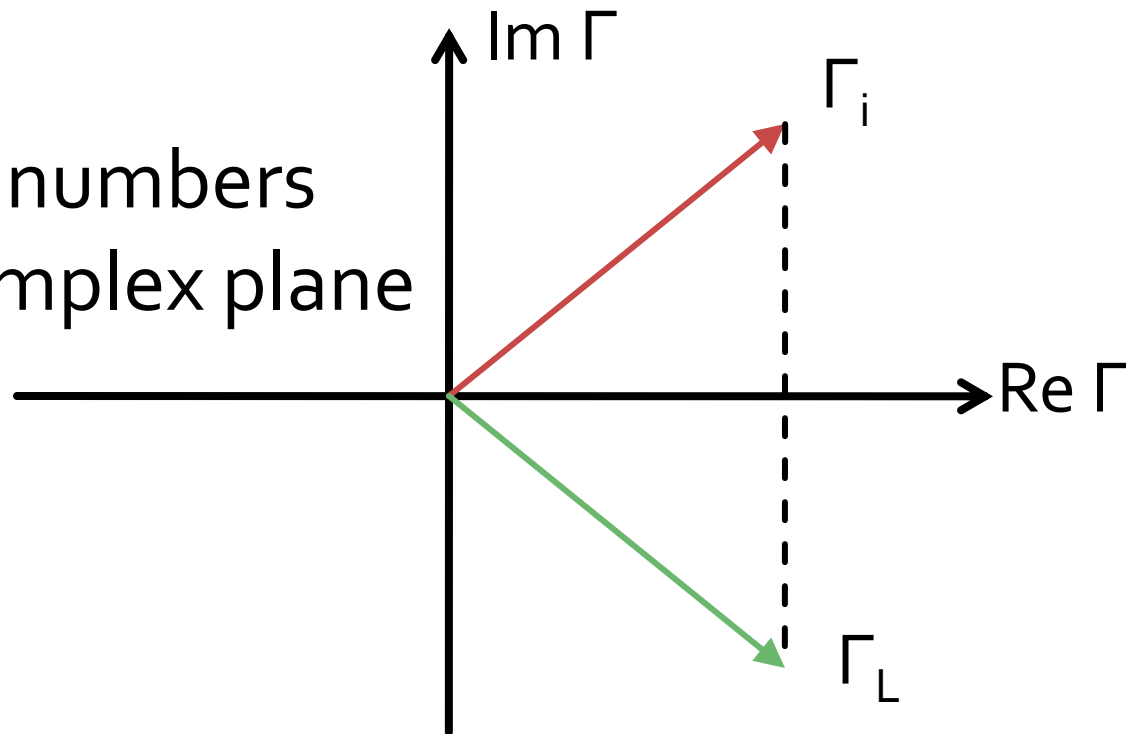
$$Z_L = Z_i^*$$

If we choose a real  $Z_0$

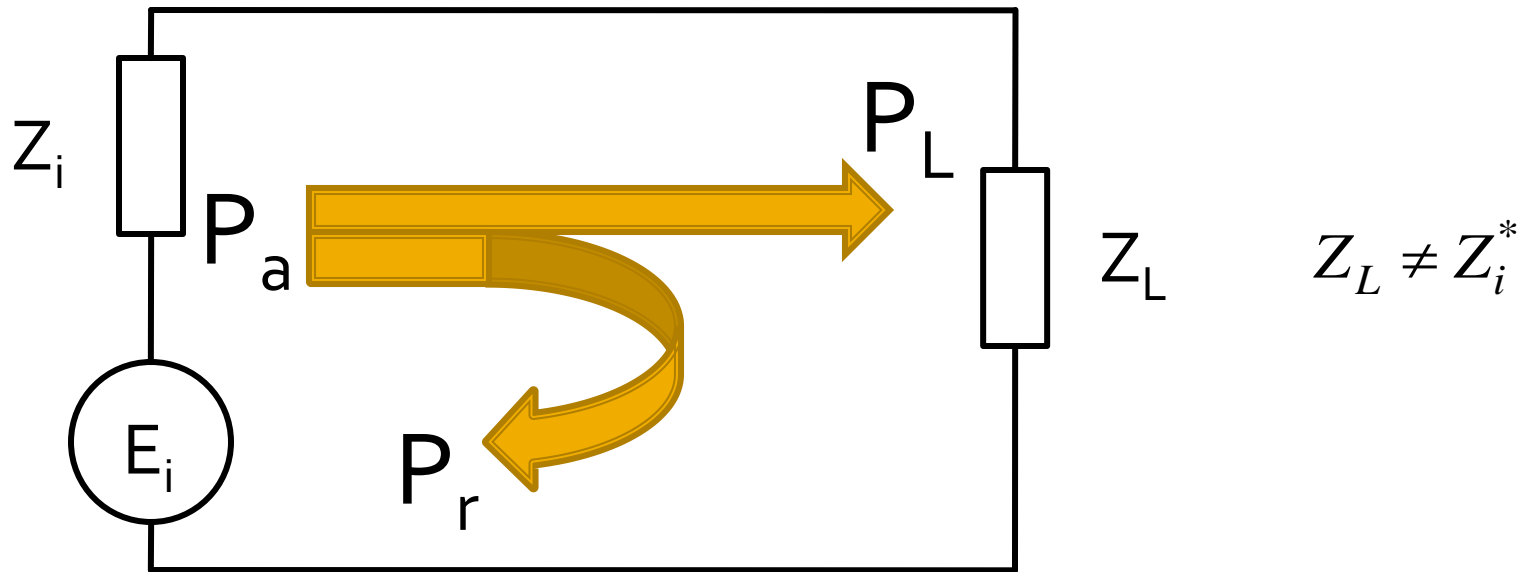
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane



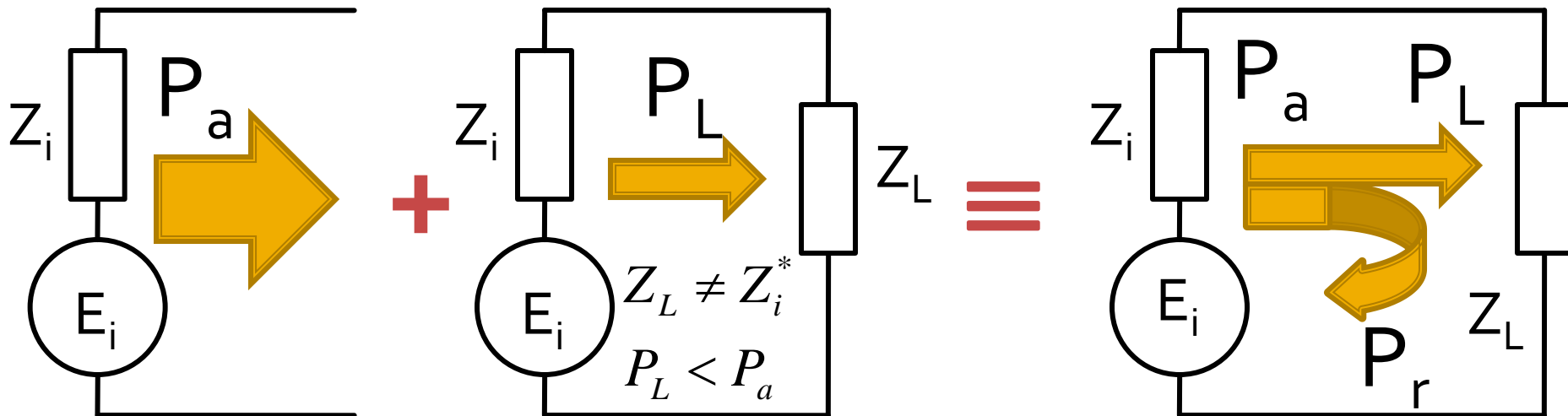
# Reflection and power / Model



- ~~Power reflection~~
- Power of the reflected wave



# Reflection and power / Model

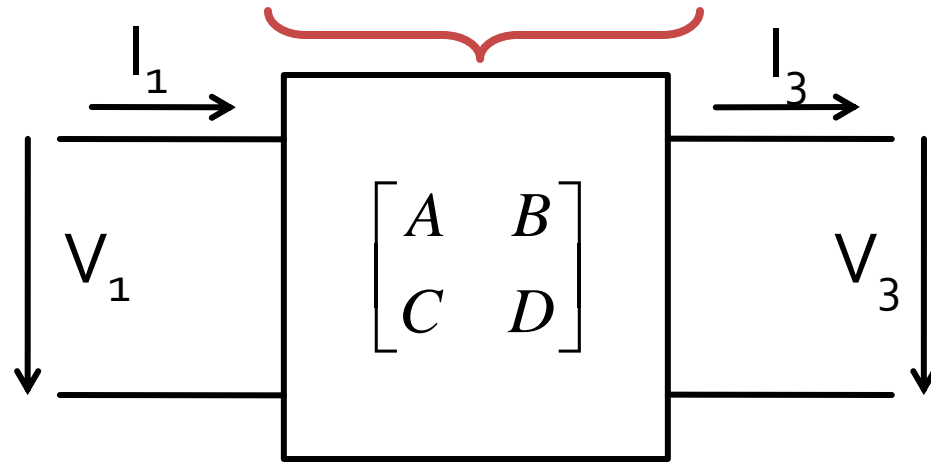
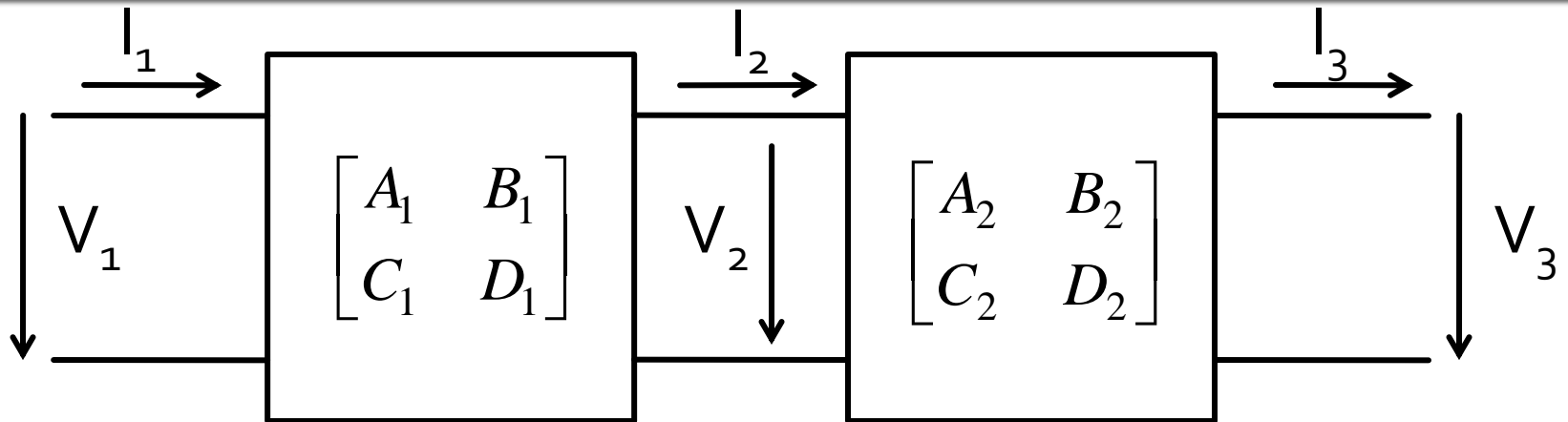


- The source has the ability to send to the load a certain maximum power (available power)  $P_a$
- For a particular load the power sent to the load is less than the maximum (mismatch)  $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected  $P_r = P_a - P_L$
- The power is a **scalar** !

Lecture 3-4

# Microwave Network Analysis

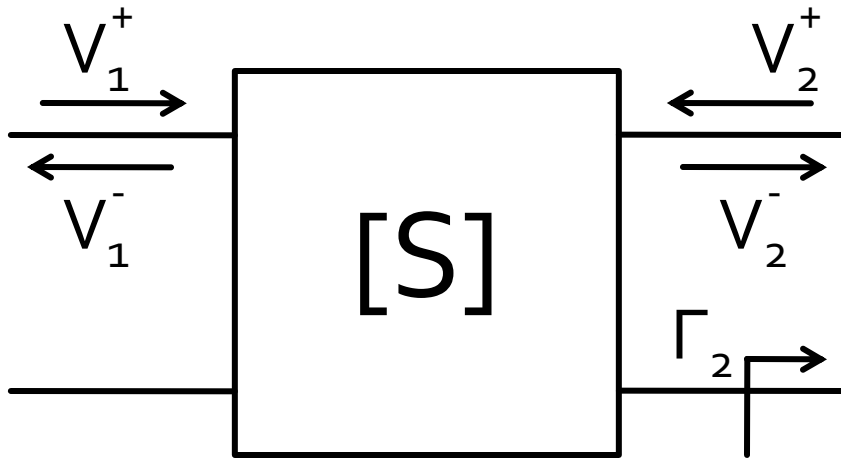
# ABCD (transmission) matrix



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

# Scattering matrix – S

- Scattering parameters



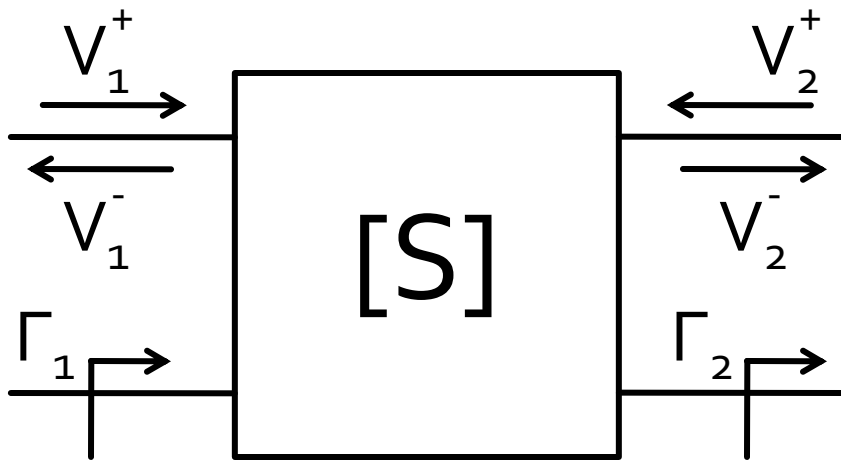
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0} \quad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0}$$

- $V_2^+ = 0$  meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

# Scattering matrix – S



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} = \Gamma_1 \Big|_{\Gamma_2=0}$$
$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0} = T_{21} \Big|_{\Gamma_2=0}$$

- $S_{11}$  is the reflection coefficient seen looking into port **1** when port **2** is terminated in matched load
- $S_{21}$  is the transmission coefficient from port **1** (**second** index) to port **2** (**first** index) when port **2** is terminated in matched load

# Scattering matrix – S

- S matrix can be extended to multiple ports

$$S_{ii} = \left. \frac{V_i^-}{V_i^+} \right|_{V_k^+ = 0, \forall k \neq i} \quad S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0, \forall k \neq j}$$

- $S_{ii}$  is the reflection coefficient seen looking into port  $i$  when all other ports are terminated in matched loads
- $S_{ij}$  is the transmission coefficient from port  $j$  (**second** index) to port  $i$  (**first** index) when all other ports are terminated in matched loads

# Properties of S matrix

- If port  $i$  is connected to a transmission line with characteristic impedance  $Z_{0i}$

$$[Z_0] = \begin{bmatrix} Z_{01} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Z_{0n} \end{bmatrix}$$

- Lecture 2  $V(z) = V_0^+ e^{-j\beta \cdot z} + V_0^- e^{j\beta \cdot z}$   $I(z) = \frac{V_0^+}{Z_0} e^{-j\beta \cdot z} - \frac{V_0^-}{Z_0} e^{j\beta \cdot z}$

In the port's reference plane,  $z=0$

$$V_i = V_i^+ + V_i^- \quad I_i = \frac{V_i^+}{Z_{0i}} - \frac{V_i^-}{Z_{0i}}$$

- Relation to Z matrix

$$[Z] \cdot [I] = [V]$$

$$[Z] \cdot [I] = [Z_0]^{-1} \cdot [Z] \cdot [V^+] - [Z_0]^{-1} \cdot [Z] \cdot [V^-] \quad [V] = [V^+] + [V^-]$$

$$[Z_0]^{-1} \cdot [Z] \cdot [V^+] - [Z_0]^{-1} \cdot [Z] \cdot [V^-] = [V^+] + [V^-] \quad ([Z] - [Z_0]) \cdot [V^+] = ([Z] + [Z_0]) \cdot [V^-]$$

$$[V^-] = [S] \cdot [V^+]$$

$$[S] = ([Z] - [Z_0]) \cdot ([Z] + [Z_0])^{-1}$$

# Generalized Scattering Parameters

- We define the power wave amplitudes  $a$  and  $b$

$$a = \frac{V + Z_R \cdot I}{2 \cdot \sqrt{R_R}} \quad \text{the incident power wave} \quad Z_R = R_R + j \cdot X_R$$

$$b = \frac{V - Z_R^* \cdot I}{2 \cdot \sqrt{R_R}} \quad \text{the reflected power wave}$$

Any complex impedance,  
named reference impedance

- Total voltage and current in terms of the power wave amplitudes

$$V = \frac{Z_R^* \cdot a + Z_R \cdot b}{\sqrt{R_R}}$$

$$I = \frac{a - b}{\sqrt{R_R}}$$



# Power waves for N ports

$$[b] = [F] \cdot ([Z] - [Z_R]^*) \cdot ([Z] + [Z_R])^{-1} \cdot [F]^{-1} \cdot [a]$$

- The scattering matrix for power waves,  $[S_p]$

$$[b] = [S_p] \cdot [a]$$

$$[S_p] = [F] \cdot ([Z] - [Z_R]^*) \cdot ([Z] + [Z_R])^{-1} \cdot [F]^{-1}$$

- But:  $[S] = ([Z] - [Z_0]) \cdot ([Z] + [Z_0])^{-1}$

- Typically

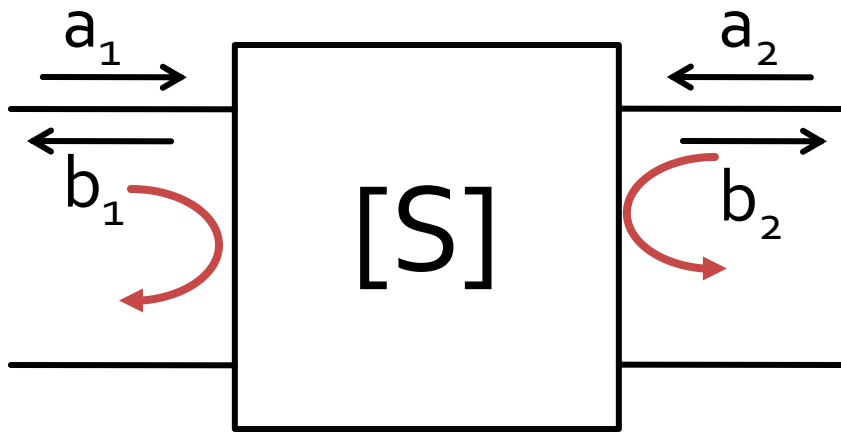
$$Z_{0i} = Z_{Ri} = R_0, \forall i$$

$$R_0 = 50\Omega$$

$$[S_p] \equiv [S]$$

- they coincide!!!

# Scattering matrix – S

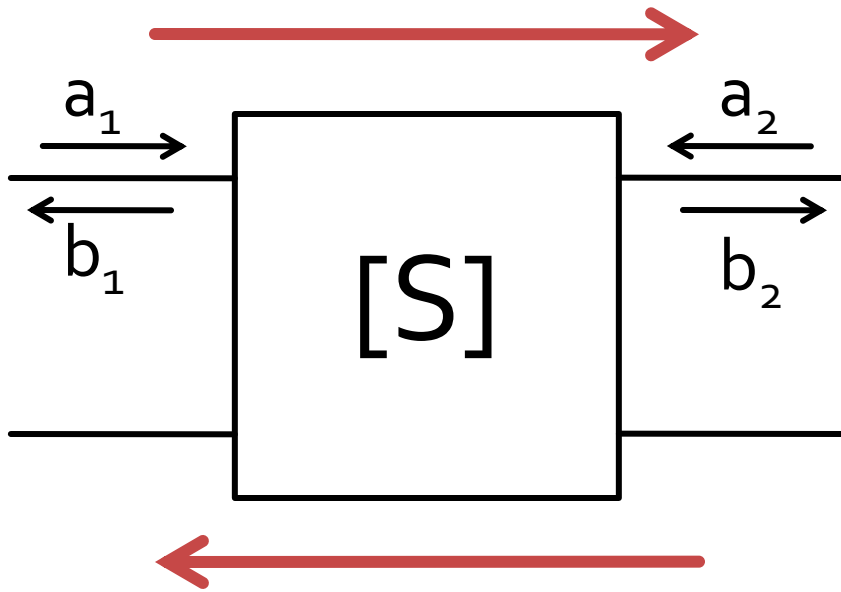


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

- $S_{11}$  and  $S_{22}$  are reflection coefficients at ports 1 and 2 when the other port is matched

# Scattering matrix – S

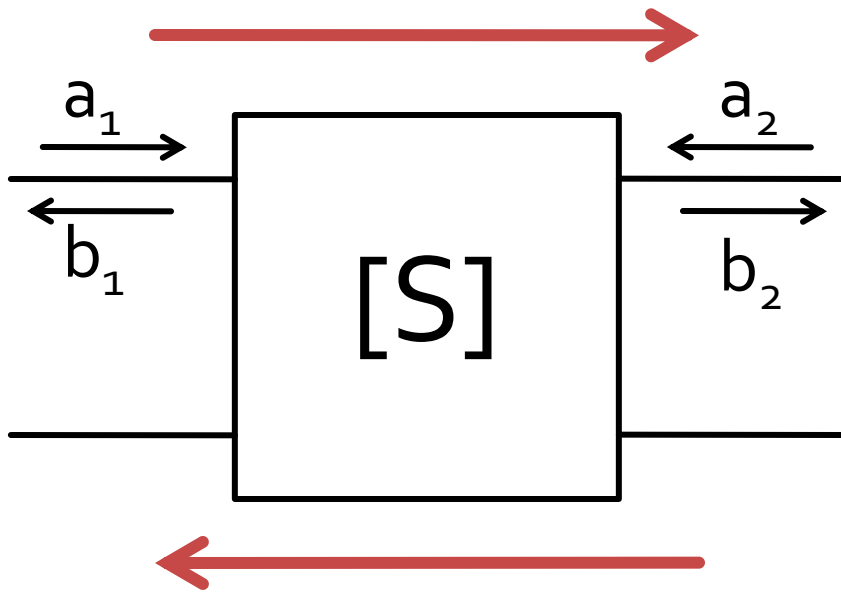


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \quad S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

- $S_{21}$  si  $S_{12}$  are signal amplitude gain when the other port is matched

# Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{\text{Power in } Z_0 \text{ load}}{\text{Power from } Z_0 \text{ source}}$$

- $a, b$ 
  - information about signal power **AND** signal phase
- $S_{ij}$ 
  - network effect (gain) over signal power **including** phase information

# Measuring S parameters - VNA

- Vector Network Analyzer

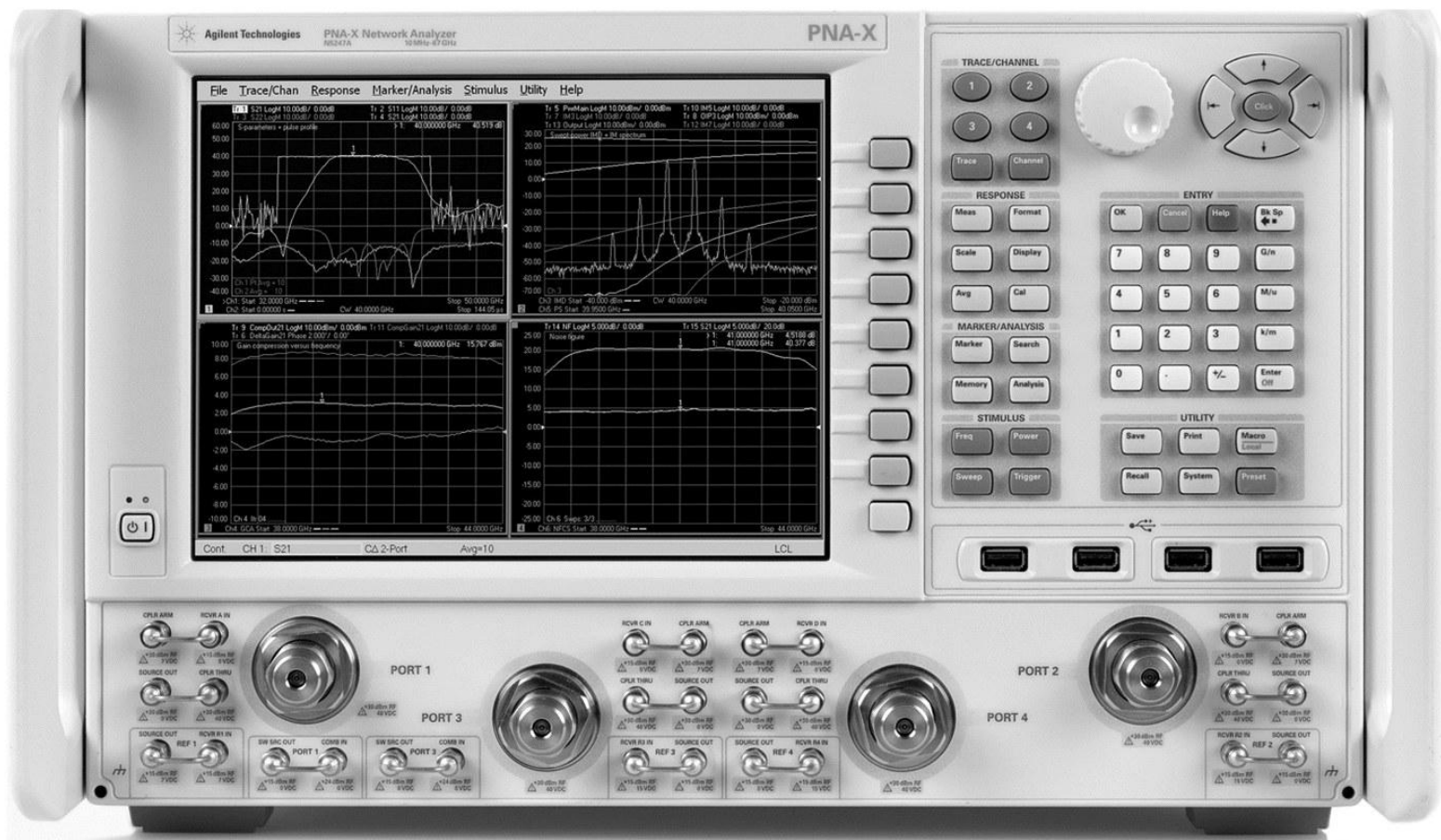


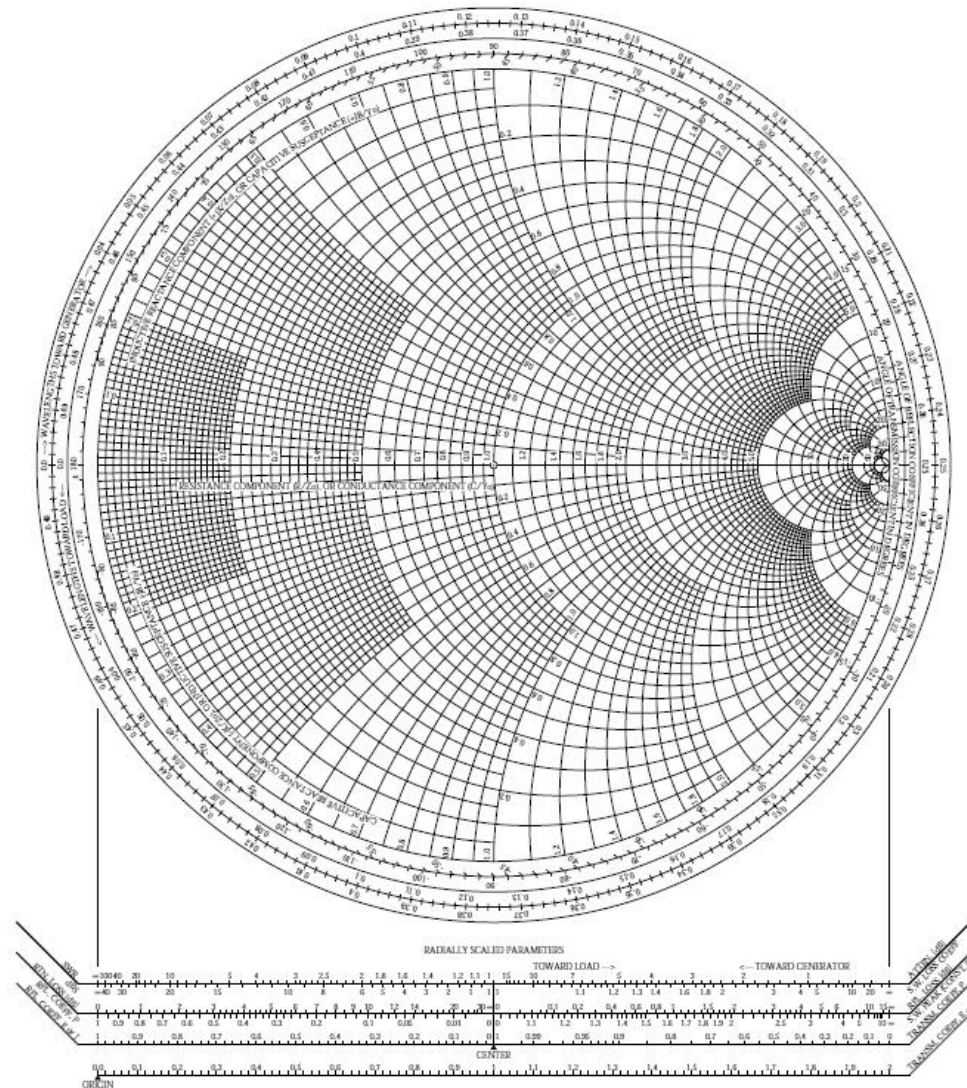
Figure 4.7  
Courtesy of Agilent Technologies

Impedance Matching

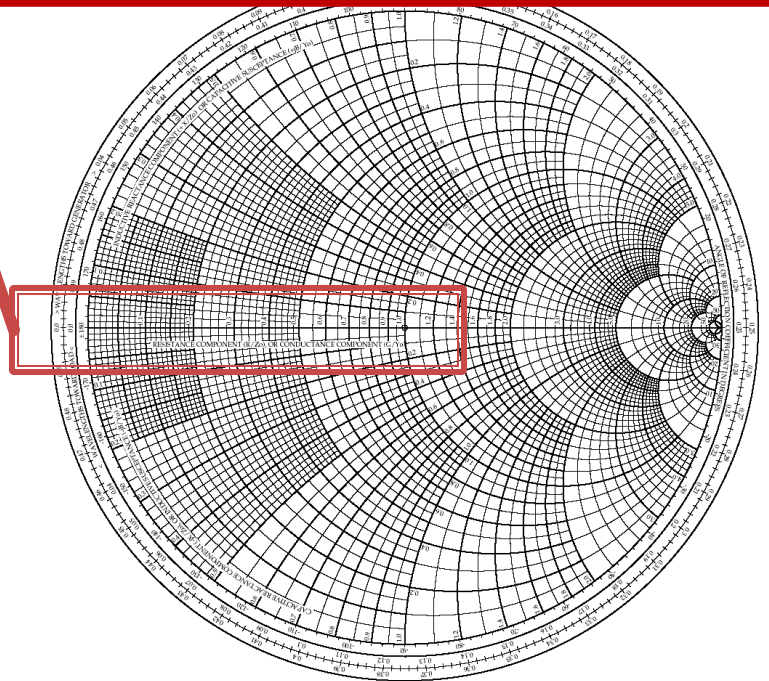
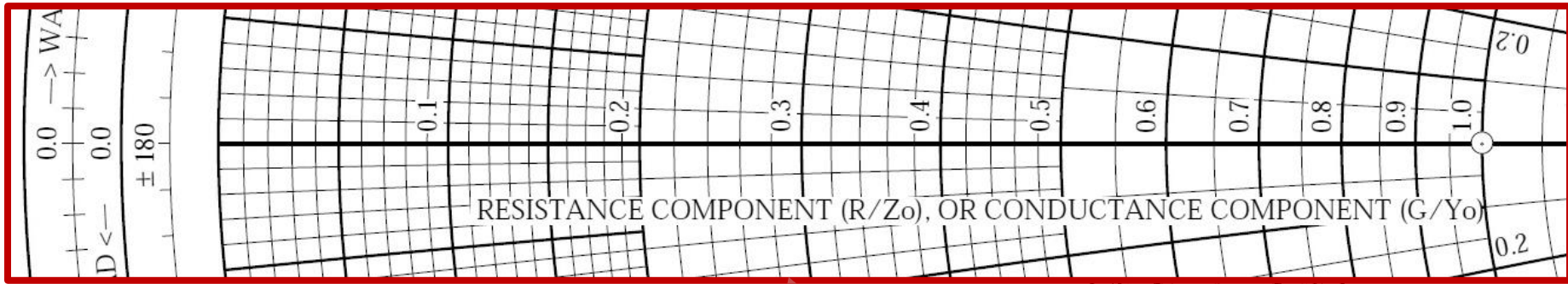
# The Smith Chart

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# The Smith Chart

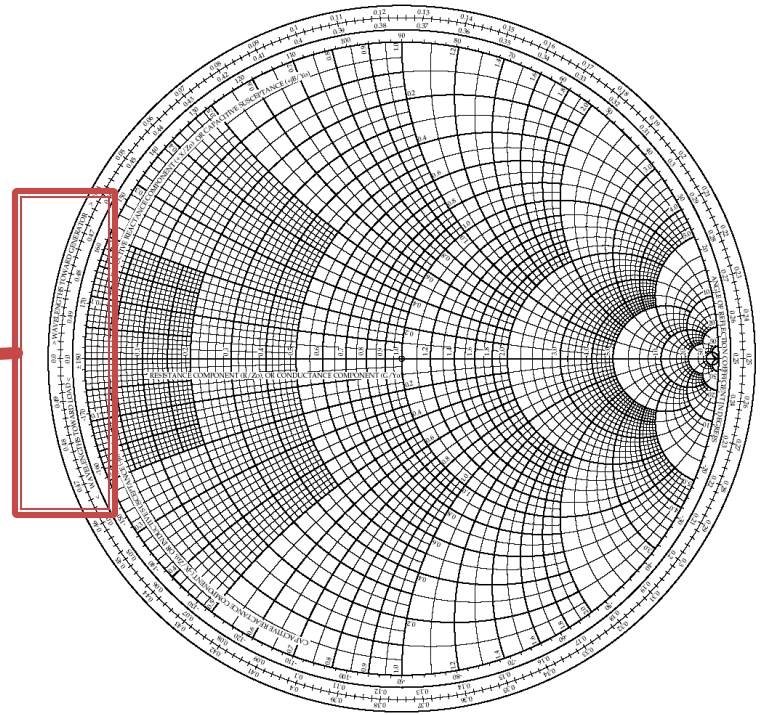
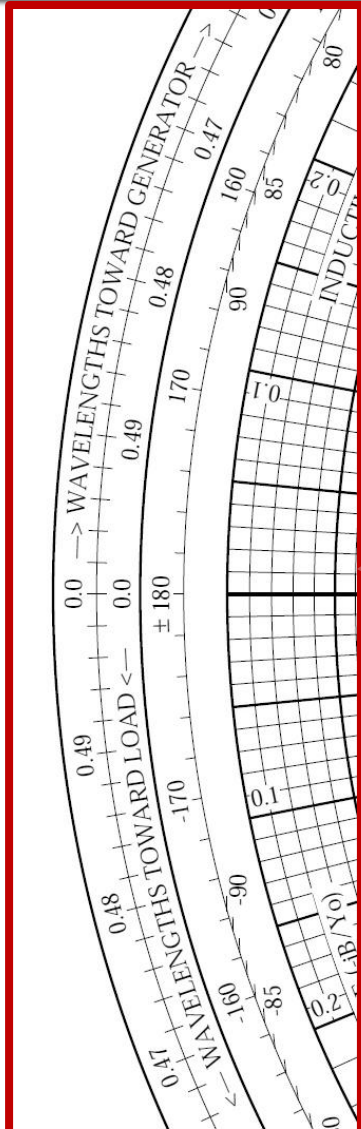


# The Smith Chart

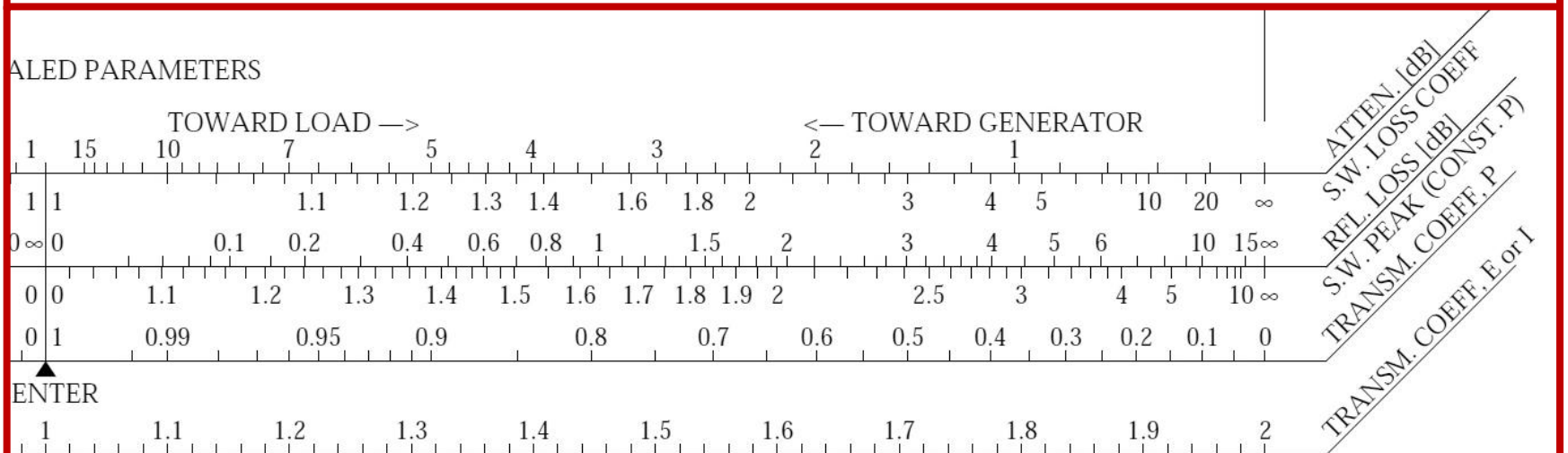
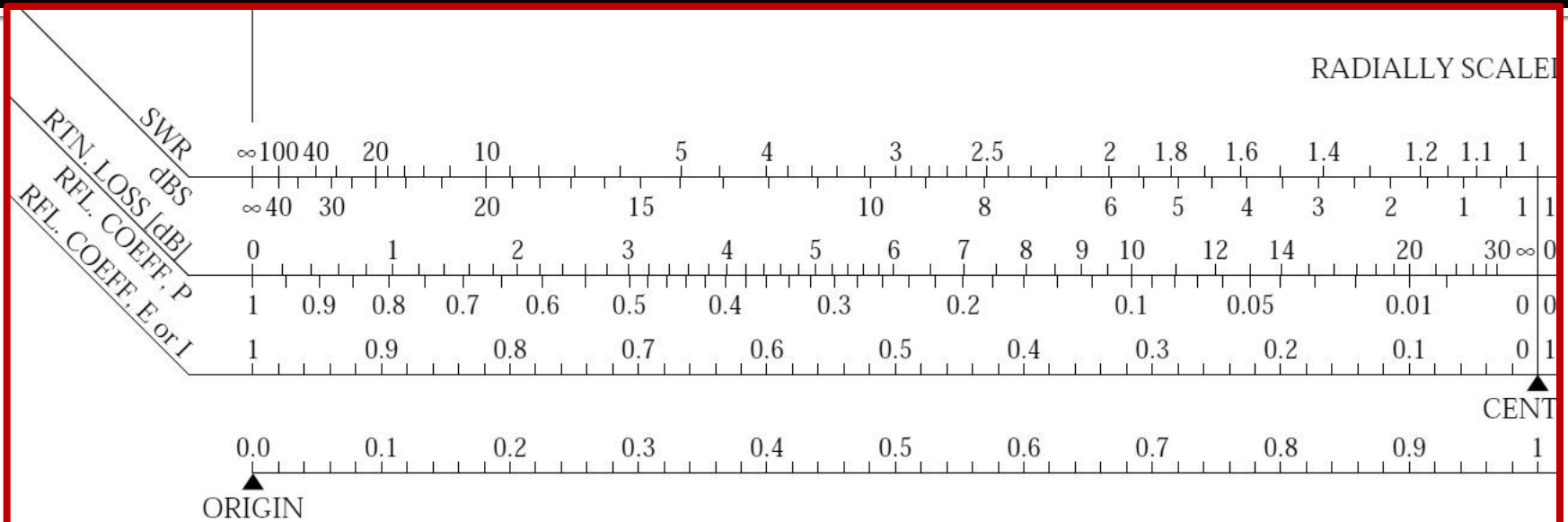




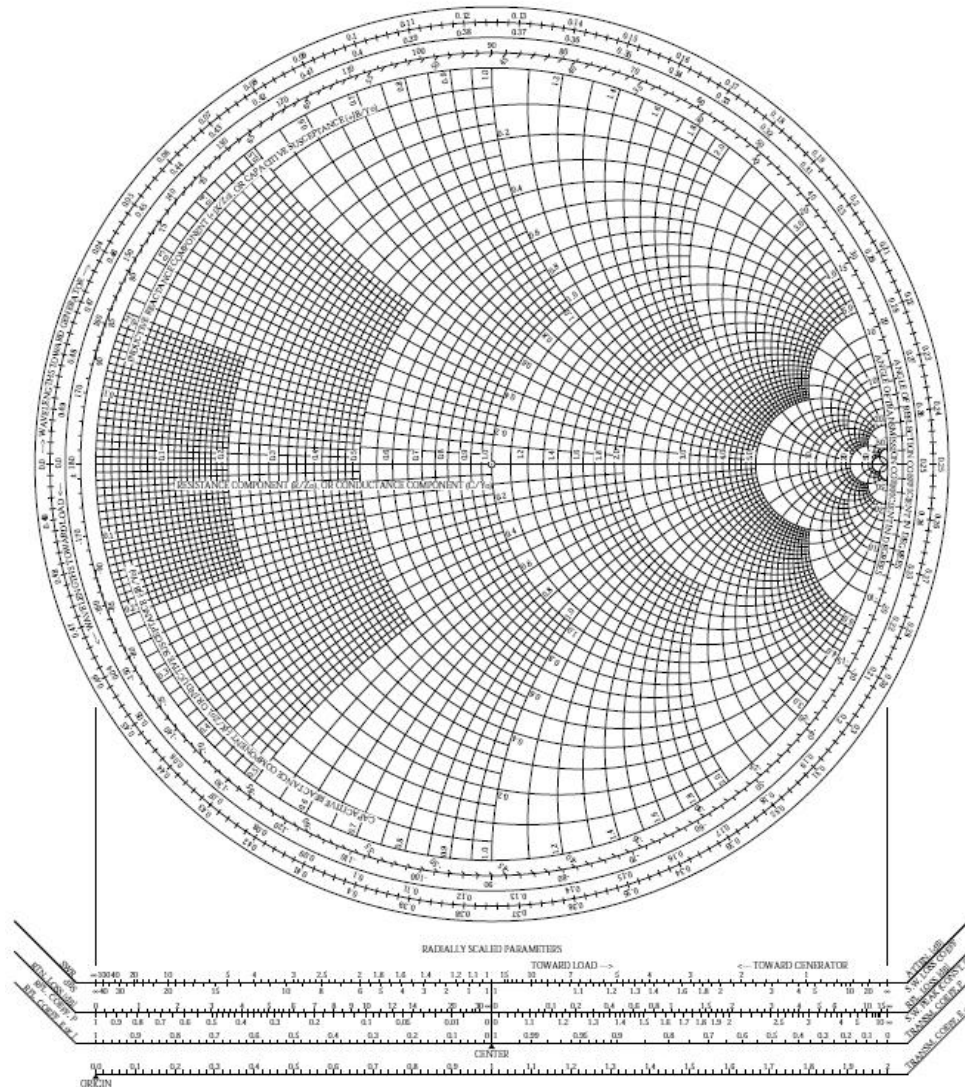
# The Smith Chart



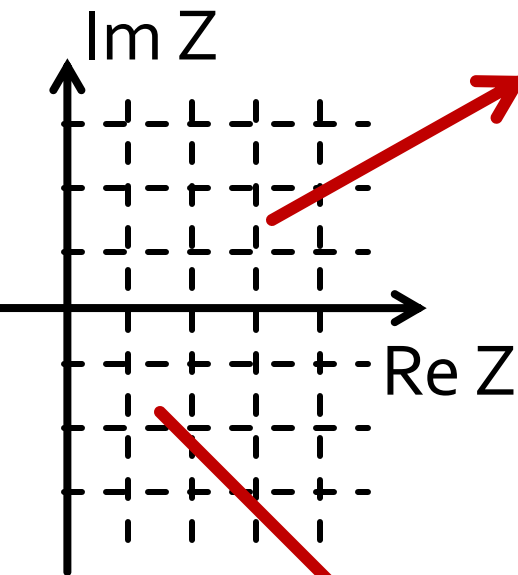
# The Smith Chart



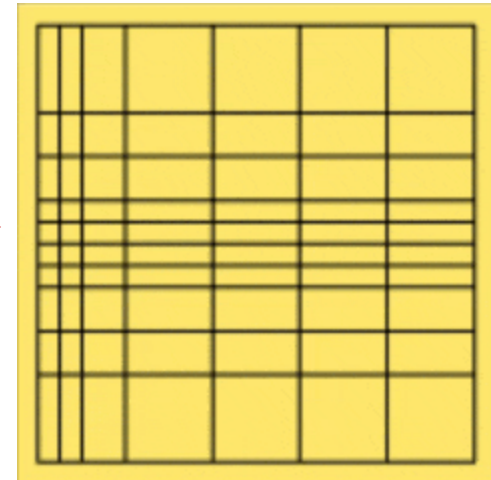
# The Smith Chart



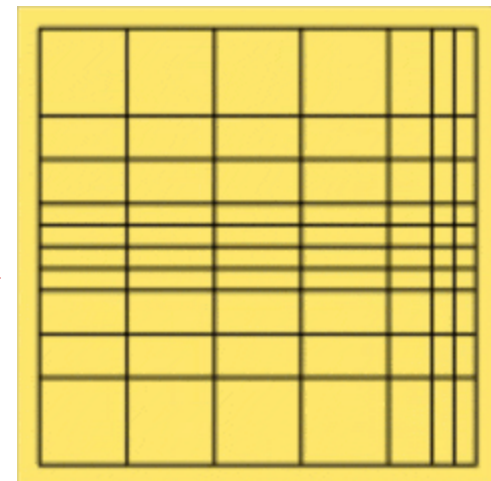
# The Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

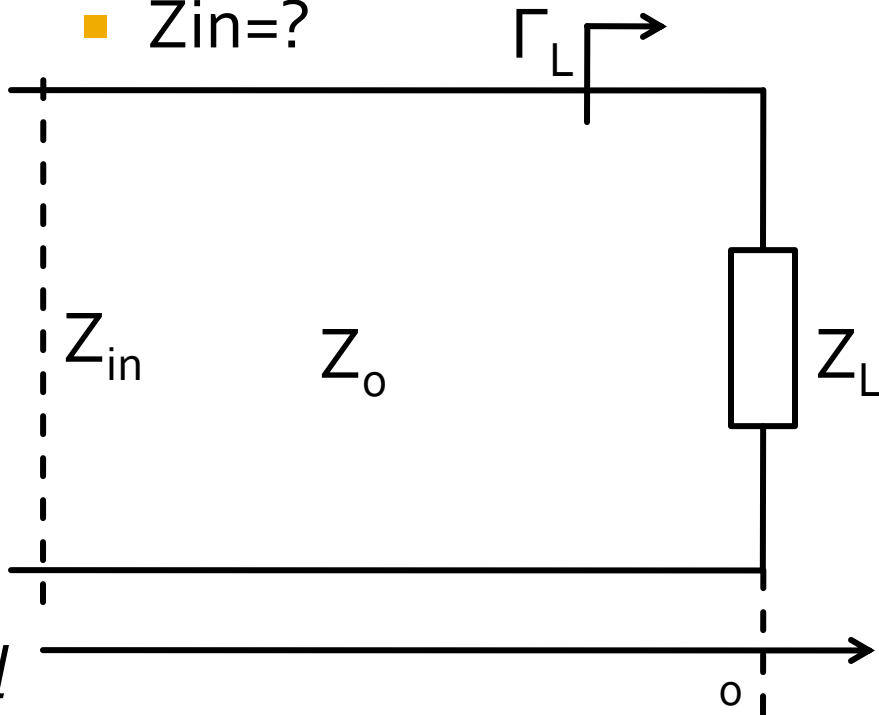


$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y_L}{1 + y_L}$$



# Traditional usage

- transmission line
  - 100Ω characteristic impedance
  - 0.3λ length
  - $Z_L = 40\Omega + j \cdot 70\Omega$  load
- $Z_{in} = ?$



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$Z_{in} = 36.5340\Omega - j \cdot 61.1190\Omega$$

# Traditional usage

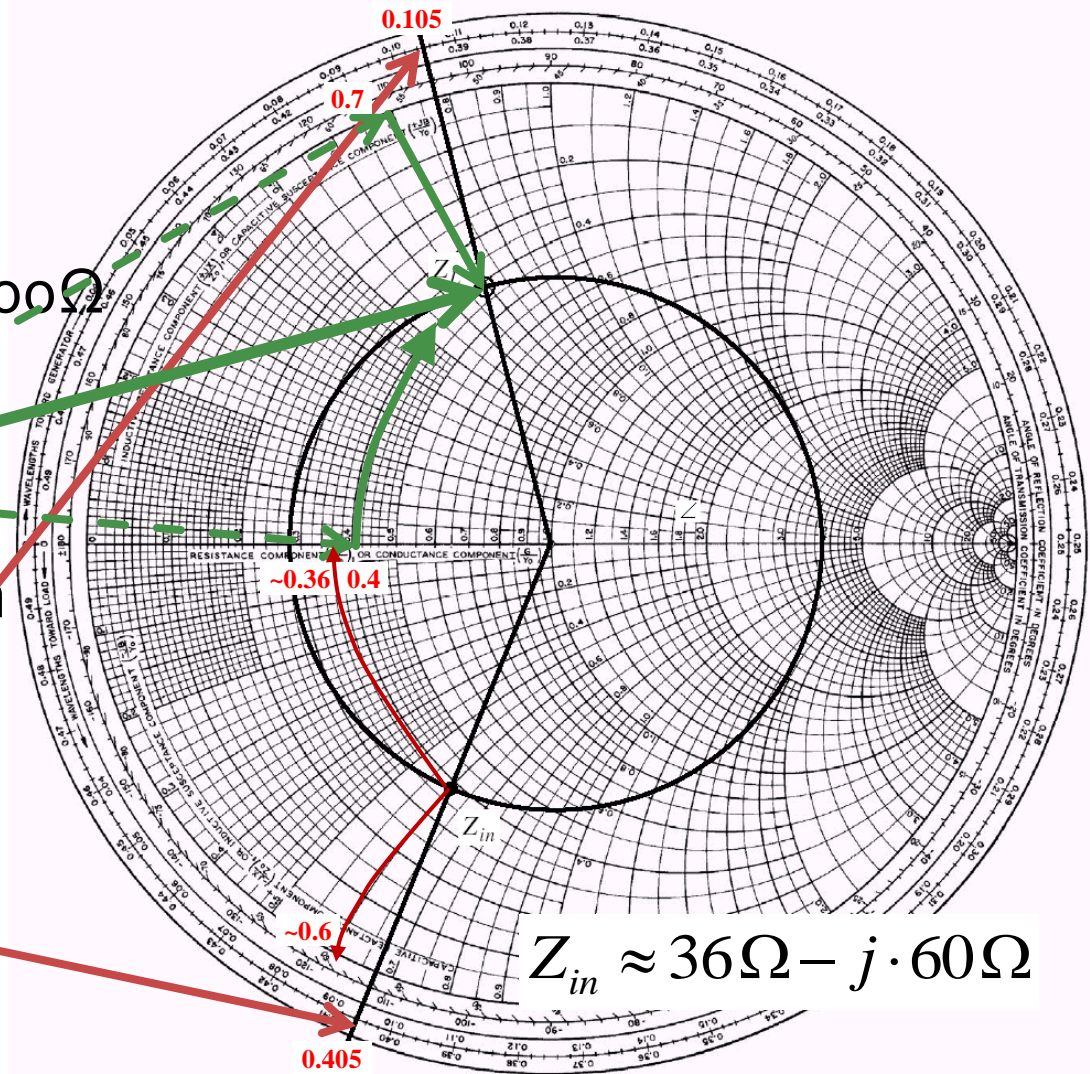
- transmission line
  - 100Ω impedance
  - 0.3λ length
  - $Z_L = 40\Omega + j \cdot 70\Omega$  load
- normalization with  $Z_0 = 100\Omega$

$$z_L = \frac{Z_L}{Z_0} = 0.4 + j \cdot 0.7$$

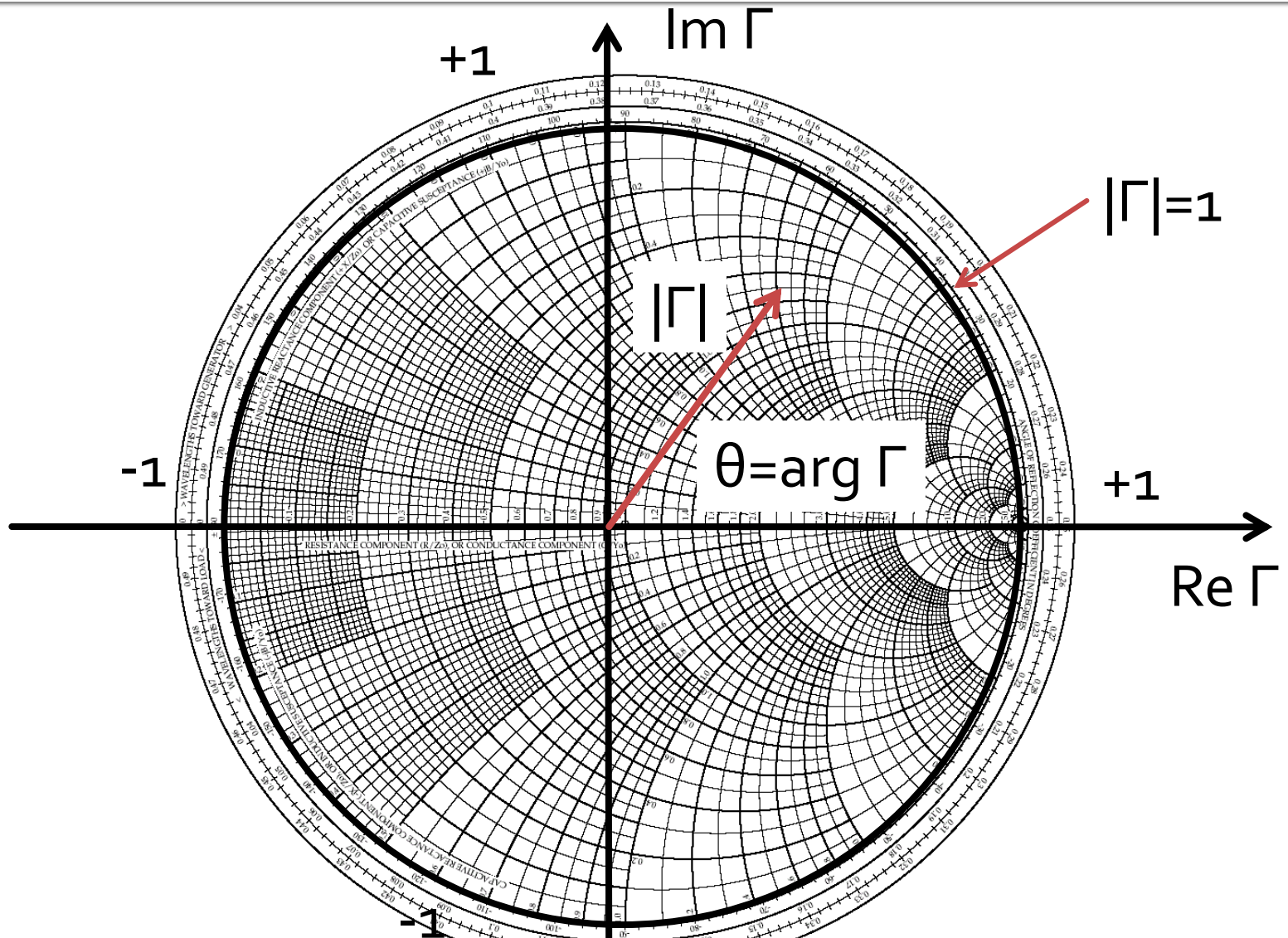
- movement with 0.3λ on a line with  $Z_0 = 100\Omega$  (**circle**)

- from  $z_L$  (0.105λ)
- to  $z_{in}$  (0.405λ)

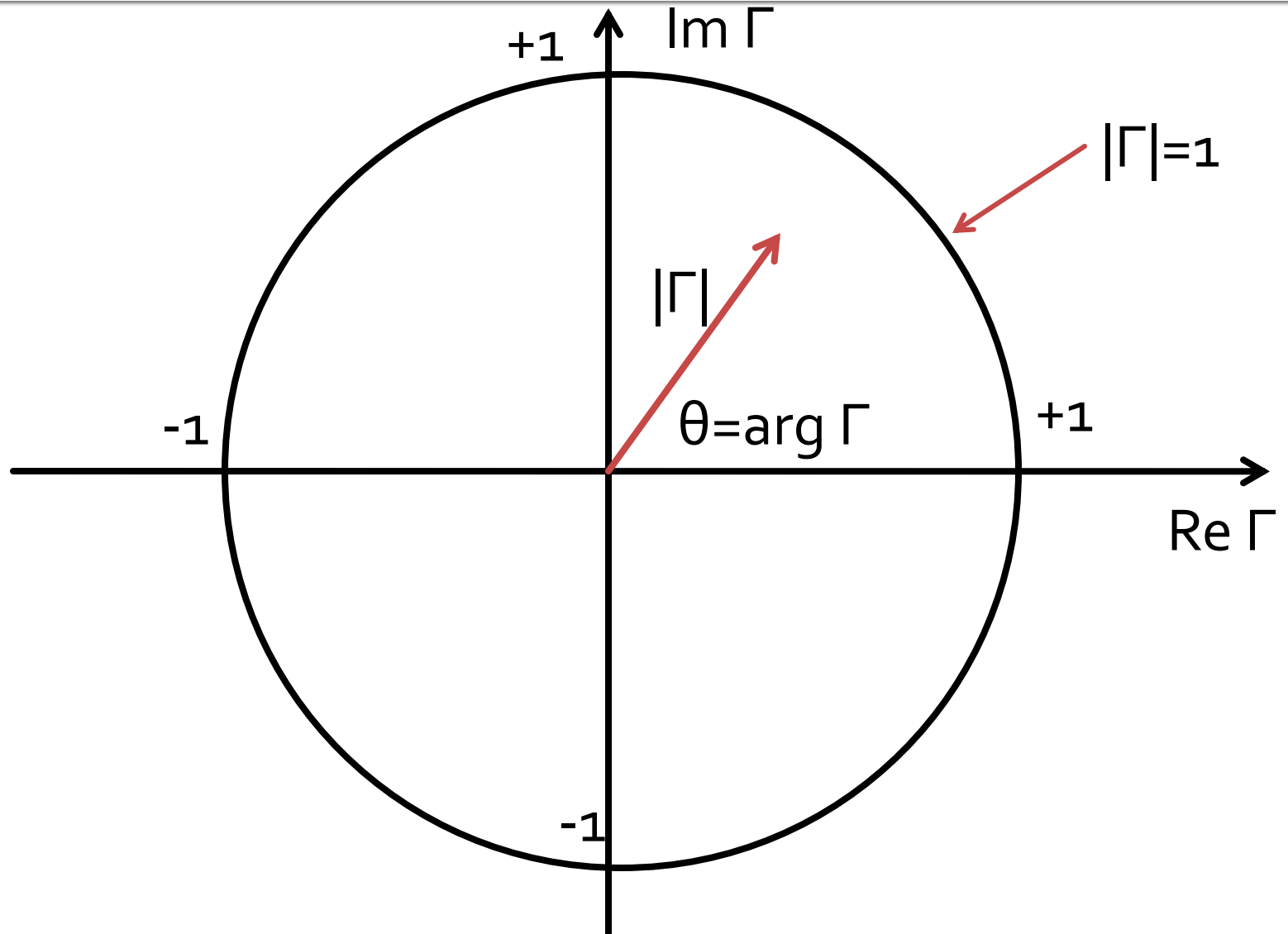
$$z_{in} \approx 0.36 - j \cdot 0.6 = \frac{Z_{in}}{Z_0}$$



# The Smith Chart



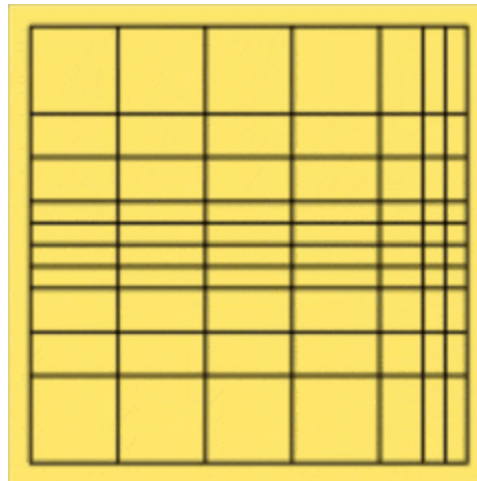
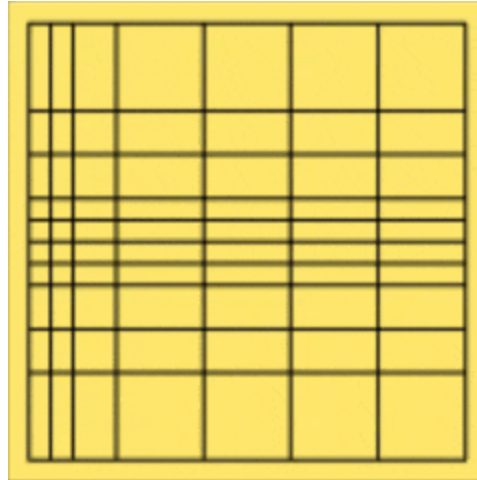
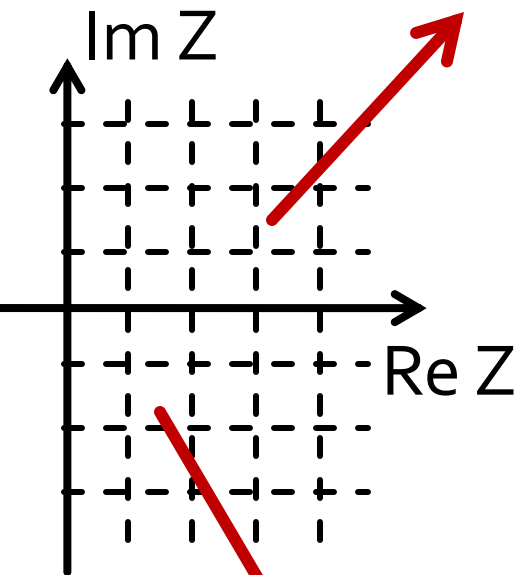
# The Smith Chart



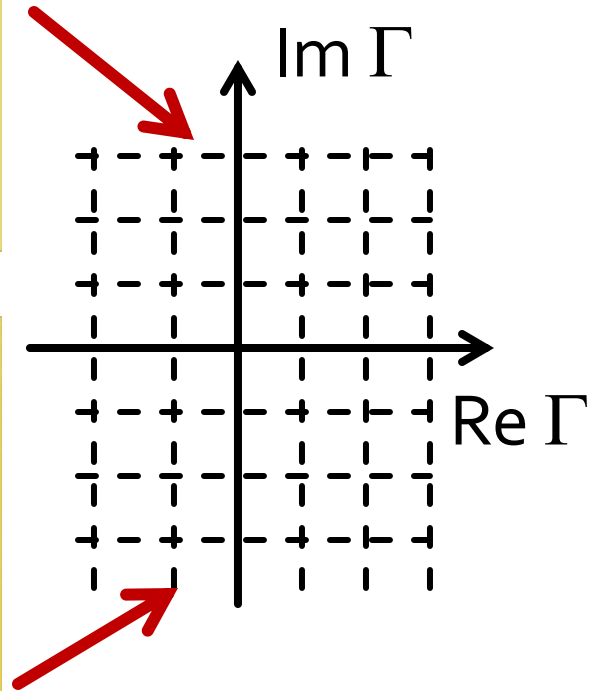


# The Smith Chart

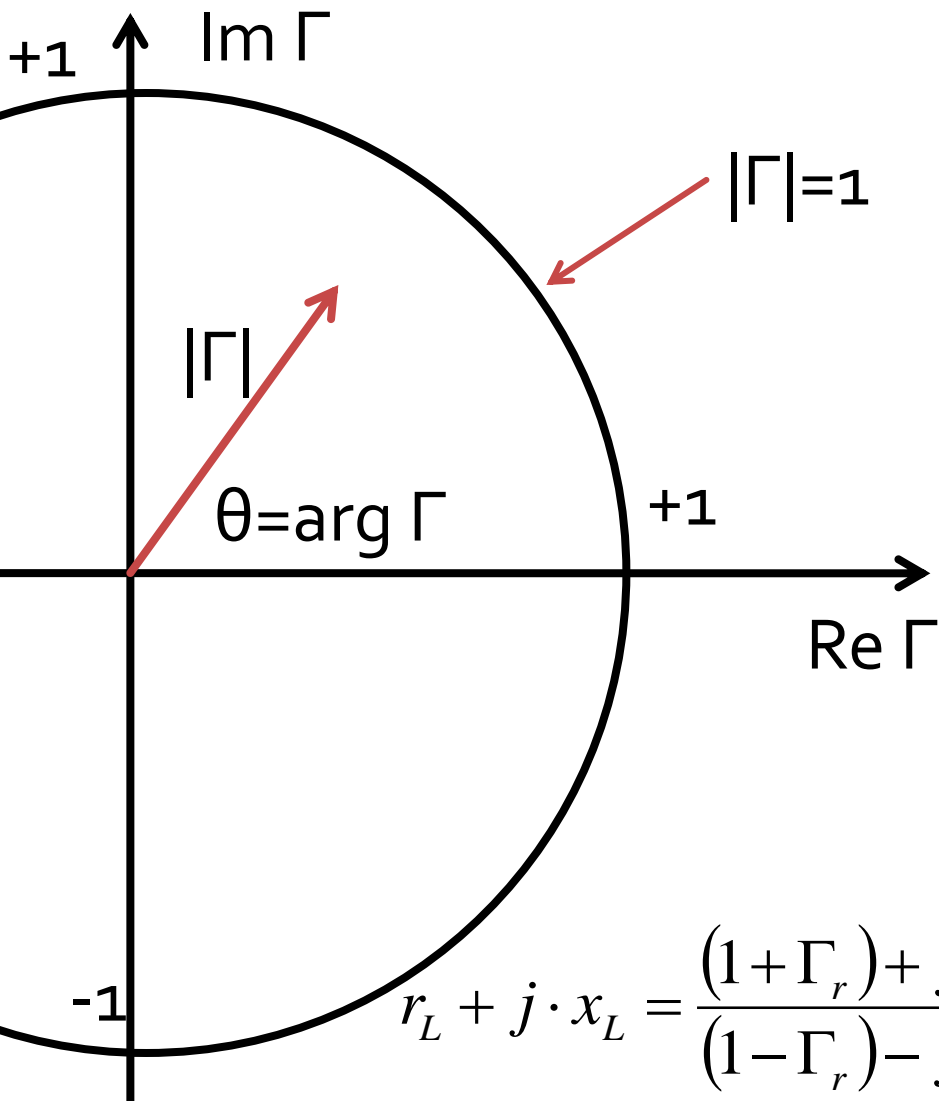
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y_L}{1 + y_L}$$



# The Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} = |\Gamma| \cdot e^{j\theta}$$

$$z_L = \frac{Z_L}{Z_0} \quad y_L = \frac{Y_L}{Y_0} = \frac{Z_0}{Z_L}$$

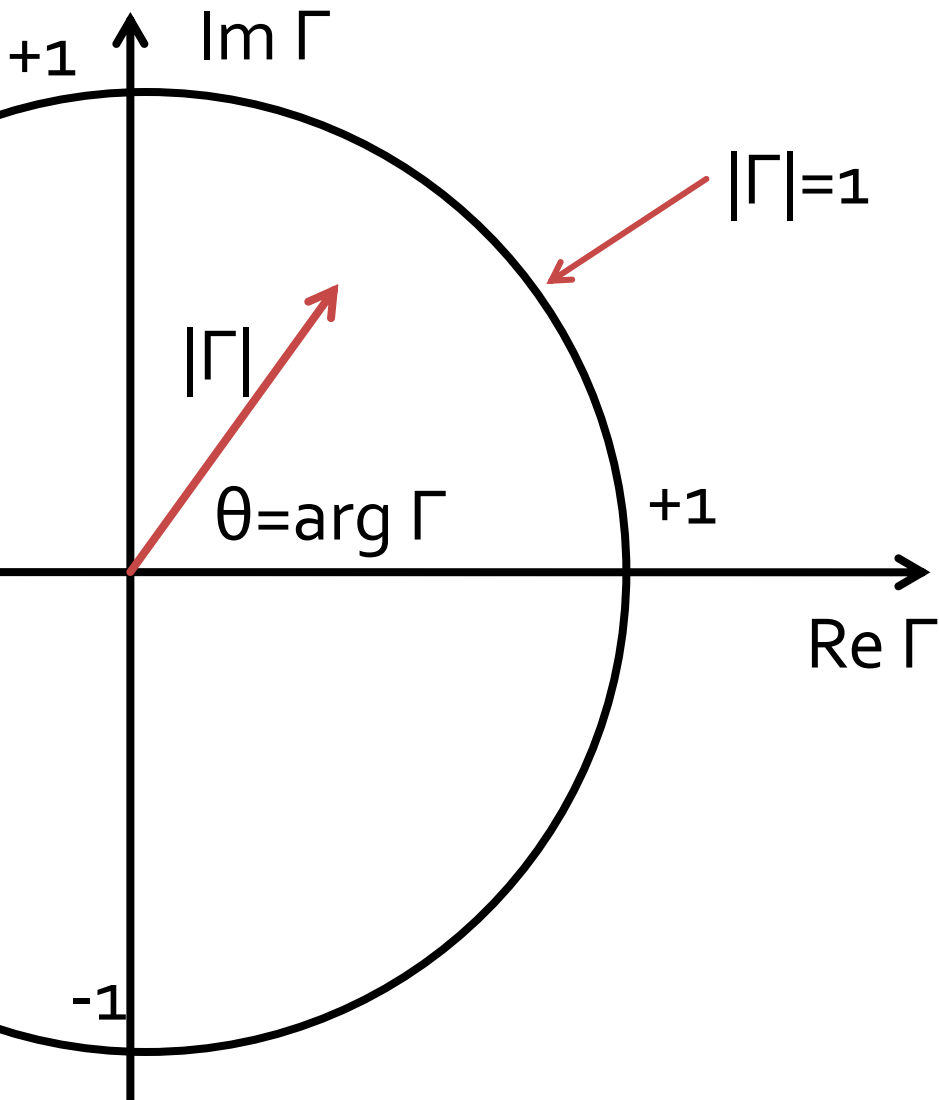
normalization  $Z_L \rightarrow z_L$  allows using the same chart for any reference impedance  $Z_0$  (the plot becomes independent of the chosen  $Z_0$ )

$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$z_L = \frac{1 + |\Gamma| \cdot e^{j\theta}}{1 - |\Gamma| \cdot e^{j\theta}} = r_L + j \cdot x_L$$

$$r_L + j \cdot x_L = \frac{(1 + \Gamma_r) + j \cdot \Gamma_i}{(1 - \Gamma_r) - j \cdot \Gamma_i} = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} + j \cdot \frac{2 \cdot \Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

# The Smith Chart



$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

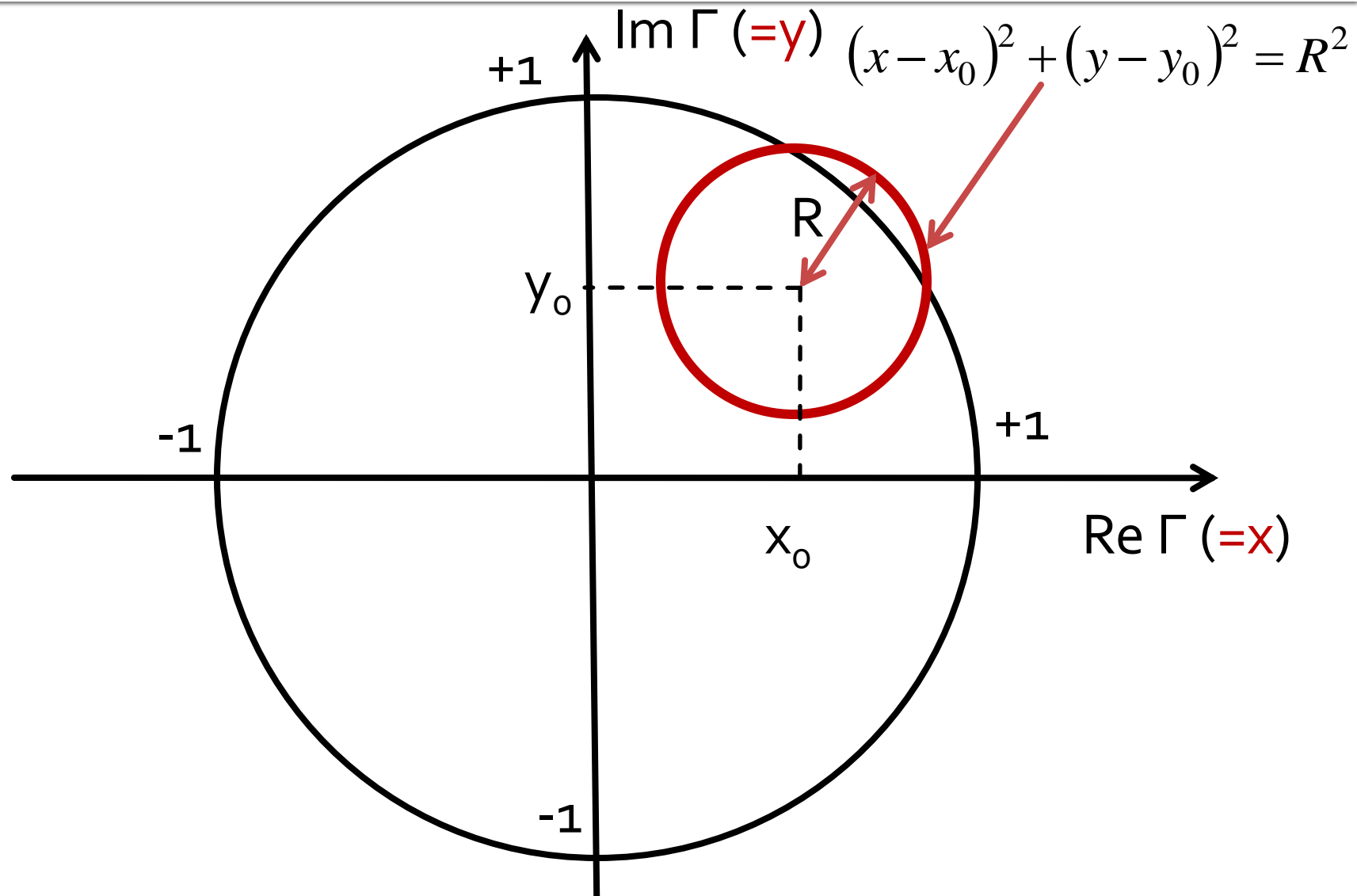
$$x_L = \frac{2 \cdot \Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

■ Rearranged

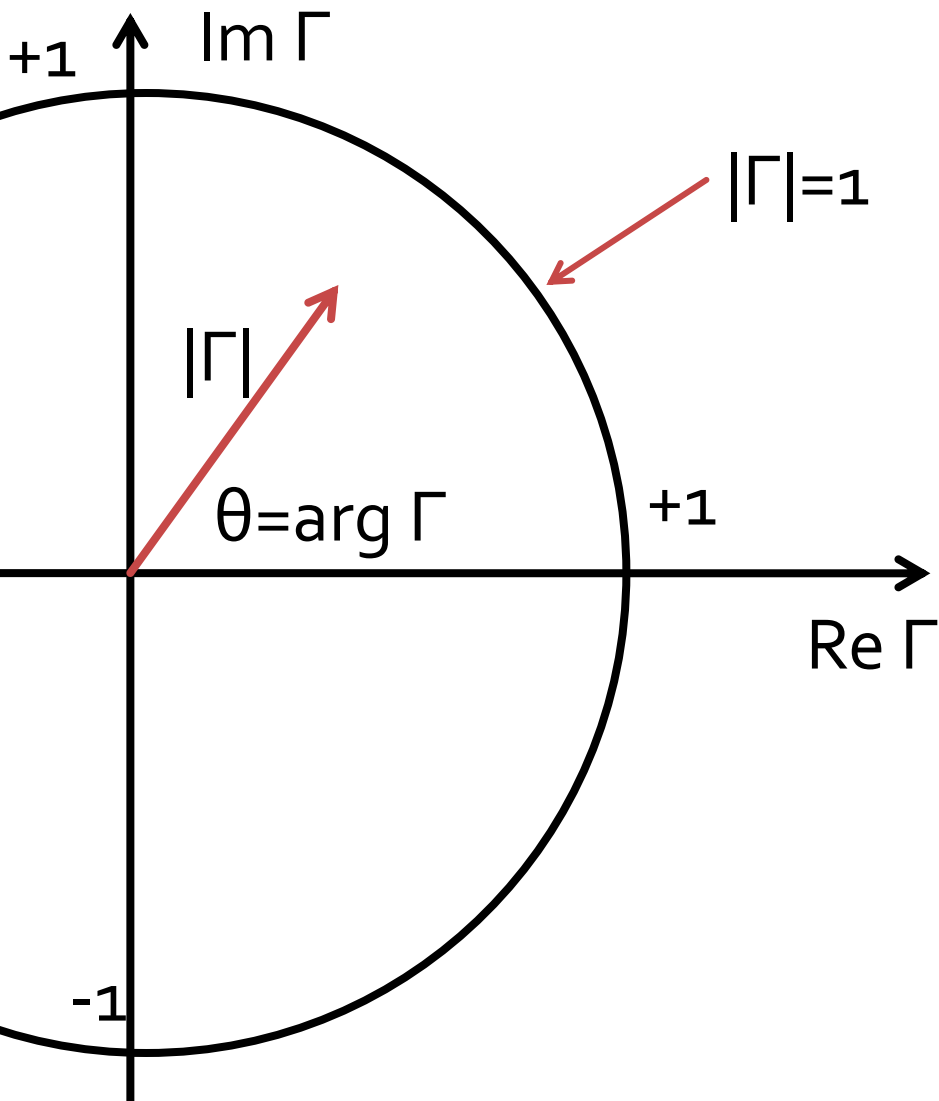
$$\left( \Gamma_r - \frac{r_L}{1 + r_L} \right)^2 + \Gamma_i^2 = \left( \frac{1}{1 + r_L} \right)^2$$

$$(\Gamma_r - 1)^2 + \left( \Gamma_i - \frac{1}{x_L} \right)^2 = \left( \frac{1}{x_L} \right)^2$$

# The Smith Chart



# The Smith Chart



$$\left( \Gamma_r - \frac{r_L}{1+r_L} \right)^2 + \Gamma_i^2 = \left( \frac{1}{1+r_L} \right)^2$$

$$(\Gamma_r - 1)^2 + \left( \Gamma_i - \frac{1}{x_L} \right)^2 = \left( \frac{1}{x_L} \right)^2$$

- **Circles** in the  $(\Gamma_r, \Gamma_i)$  complex plane

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

# The Smith Chart, resistance

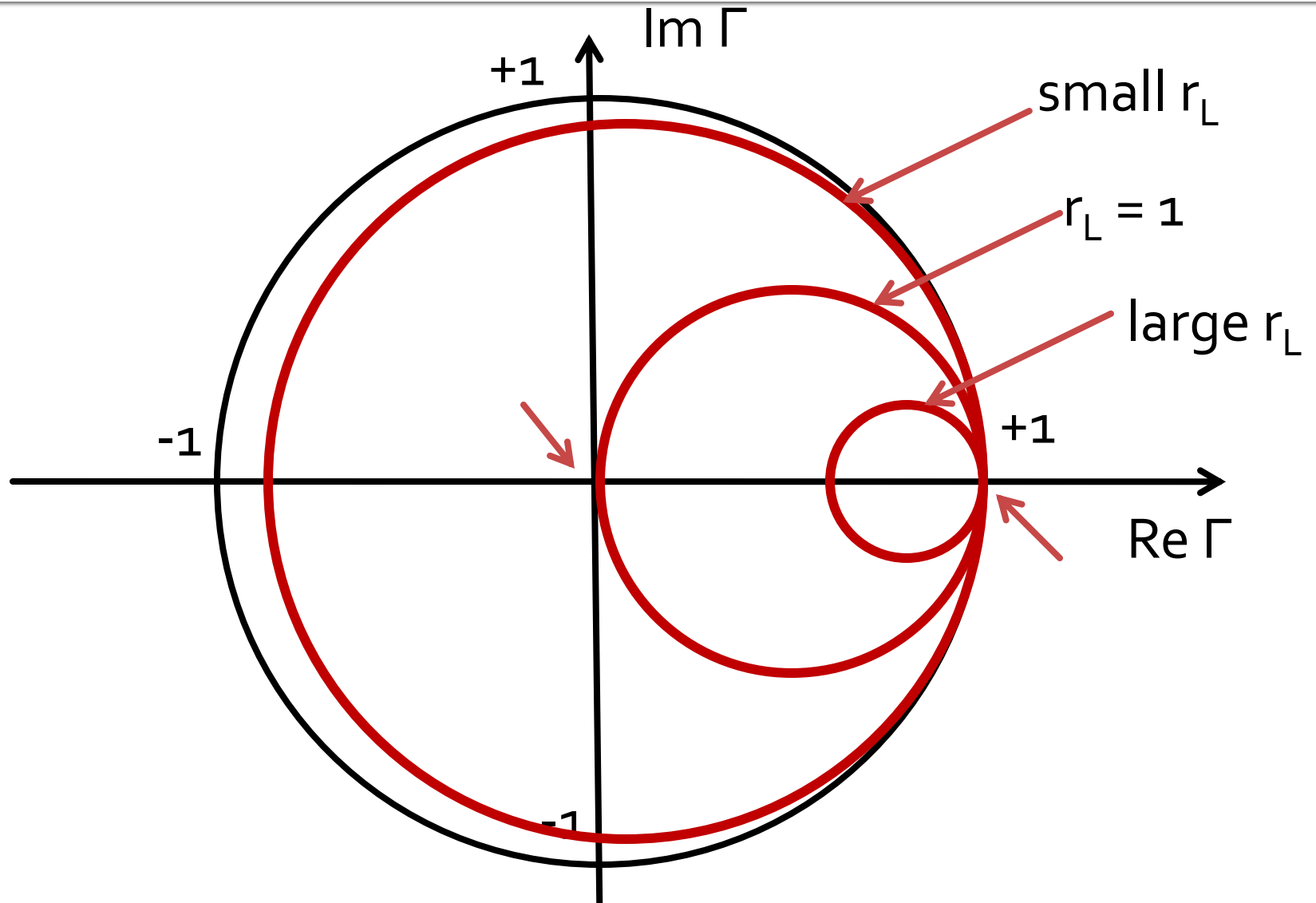
$$\left( \Gamma_r - \frac{r_L}{1+r_L} \right)^2 + \Gamma_i^2 = \left( \frac{1}{1+r_L} \right)^2$$

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$\begin{cases} x_0 = \frac{r_L}{1+r_L} \\ y_0 = 0 \\ R = \frac{1}{1+r_L} \end{cases}$$

- The locus (the set of all points whose location satisfies one or more specified conditions) of the points generated by all impedances having normalized resistance  $r_L$  is a circle which:
  - have its **center on the horizontal axis** ( $y_0=0$ )  $\left(1 - \frac{r_L}{1+r_L}\right)^2 + 0 = \left(\frac{1}{1+r_L}\right)^2$
  - passes through  **$x=1, y=0$**  point, whatever  $x_0, r_L$
  - have its radius between 0 and 1
    - tends to 0 for large  $r_L$
    - tends to 1 for small  $r_L$
  - when  $r_L$  is **1** passes also through **origin**  $\left(0 - \frac{r_L}{1+r_L}\right)^2 = \left(\frac{1}{1+r_L}\right)^2 \leftrightarrow r_L = 1$

# The Smith Chart, resistance



# The Smith Chart, reactance

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$\left\{ \begin{array}{l} x_0 = 1 \\ y_0 = \frac{1}{x_L} \\ R = \frac{1}{x_L} \end{array} \right.$$

- The locus of the points generated by all impedances having normalized resistance  $x_L$  is a circle which:

- have its **center on a line parallel with the vertical axis** ( $x_0=1$ )

- passes through  **$x=1, y=0$**  point, whatever  $x_0, x_L$

- have its radius between 0 and  $\infty$

- tends to 0 for large  $|x_L|$

- tends to  $\infty$  for small  $|x_L|$

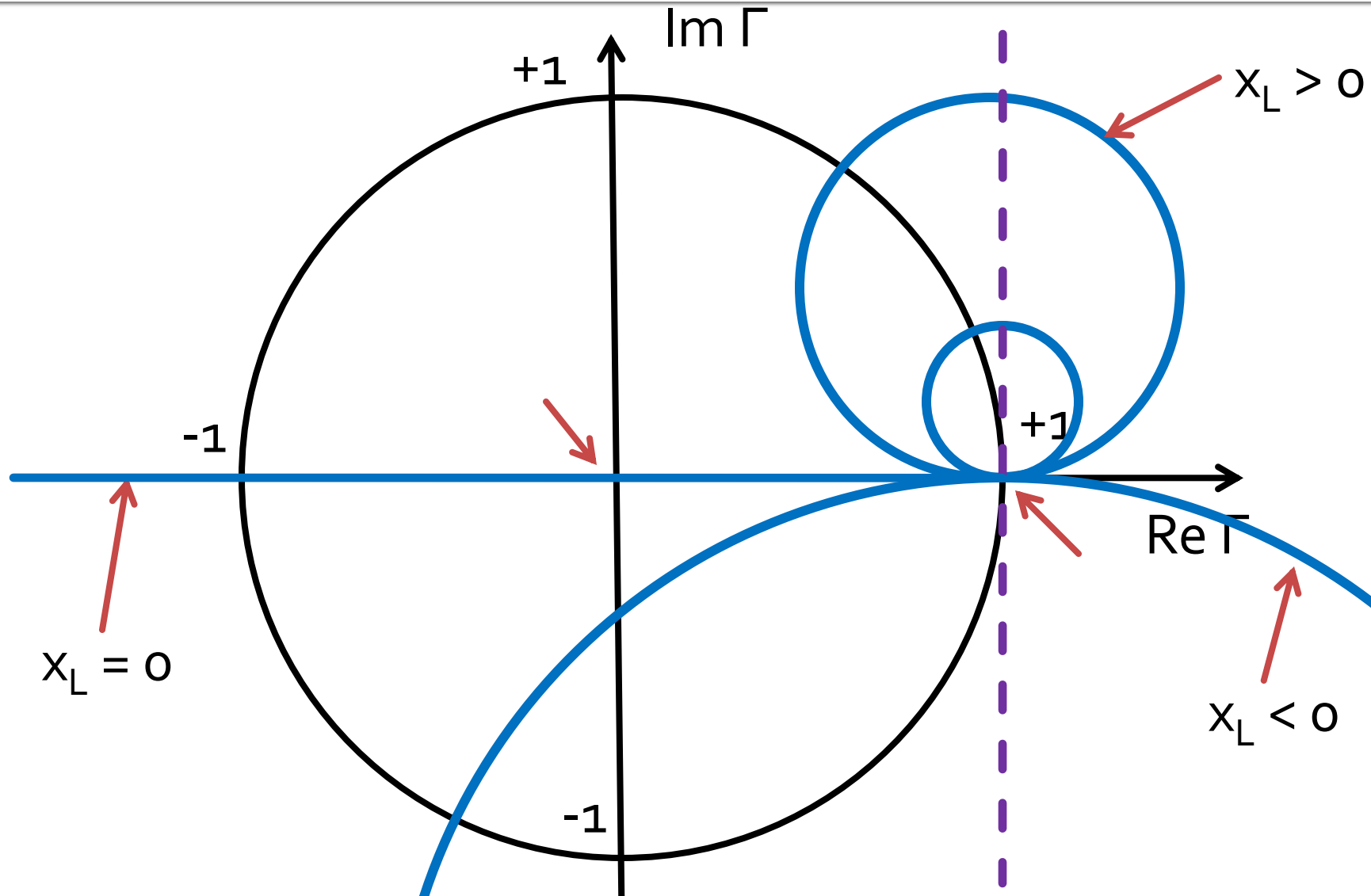
$$0 + \left(0 - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

- when  $x_L$  is **0** transforms itself in the **horizontal axis**

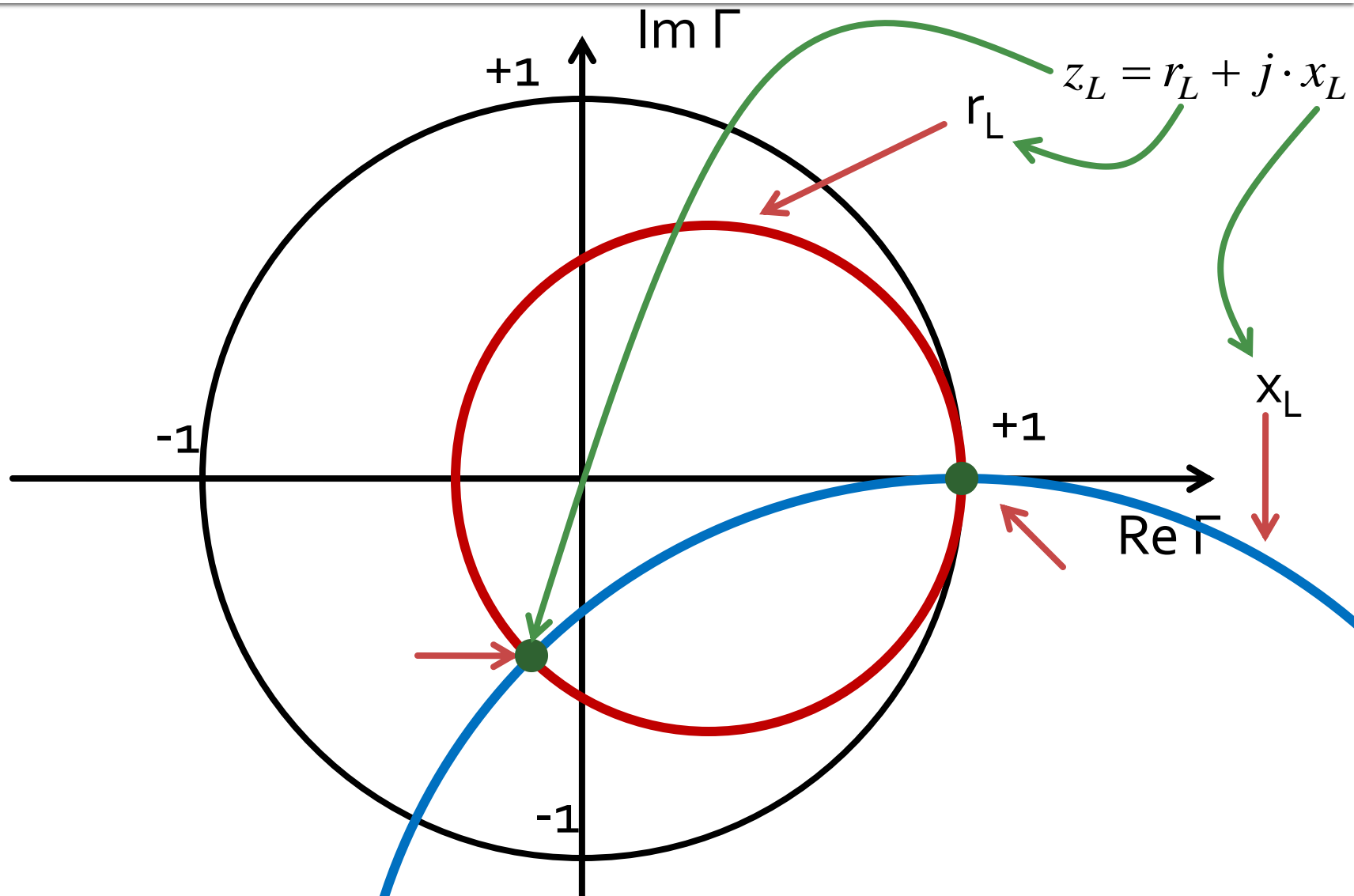
- if  $x_L > 0$  the circle is above the horizontal axis, otherwise is below it



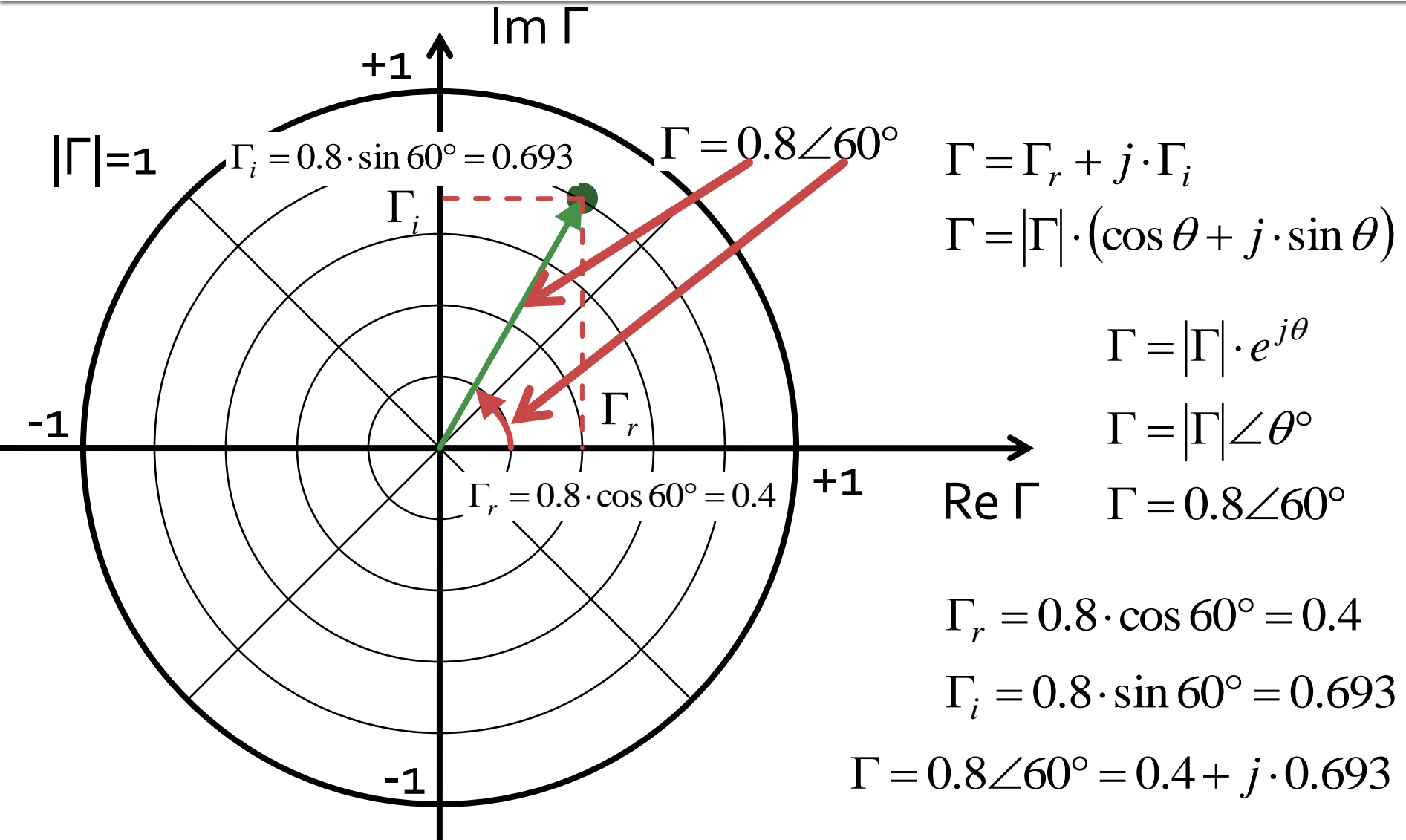
# The Smith Chart, reactance



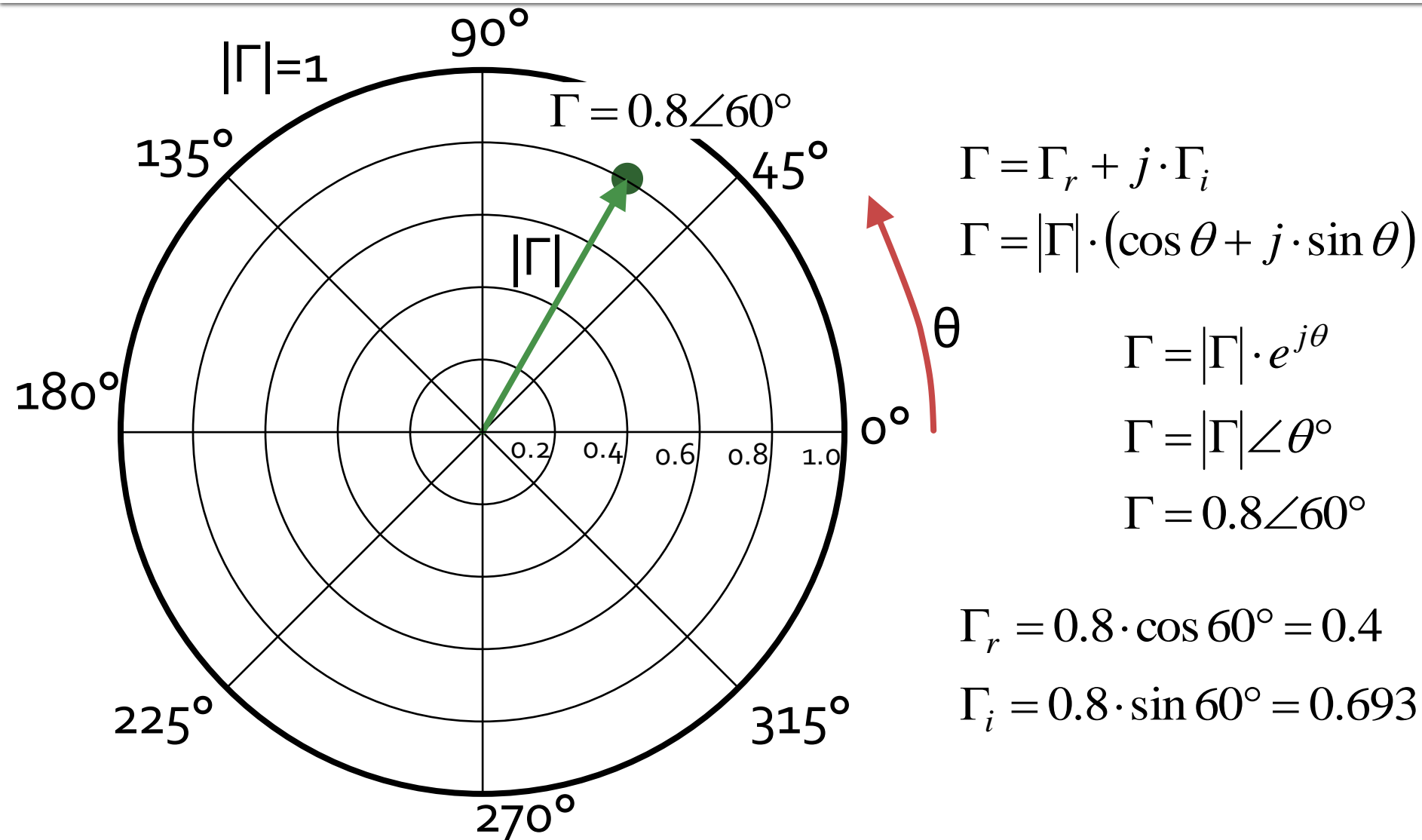
# The Smith Chart, impedance



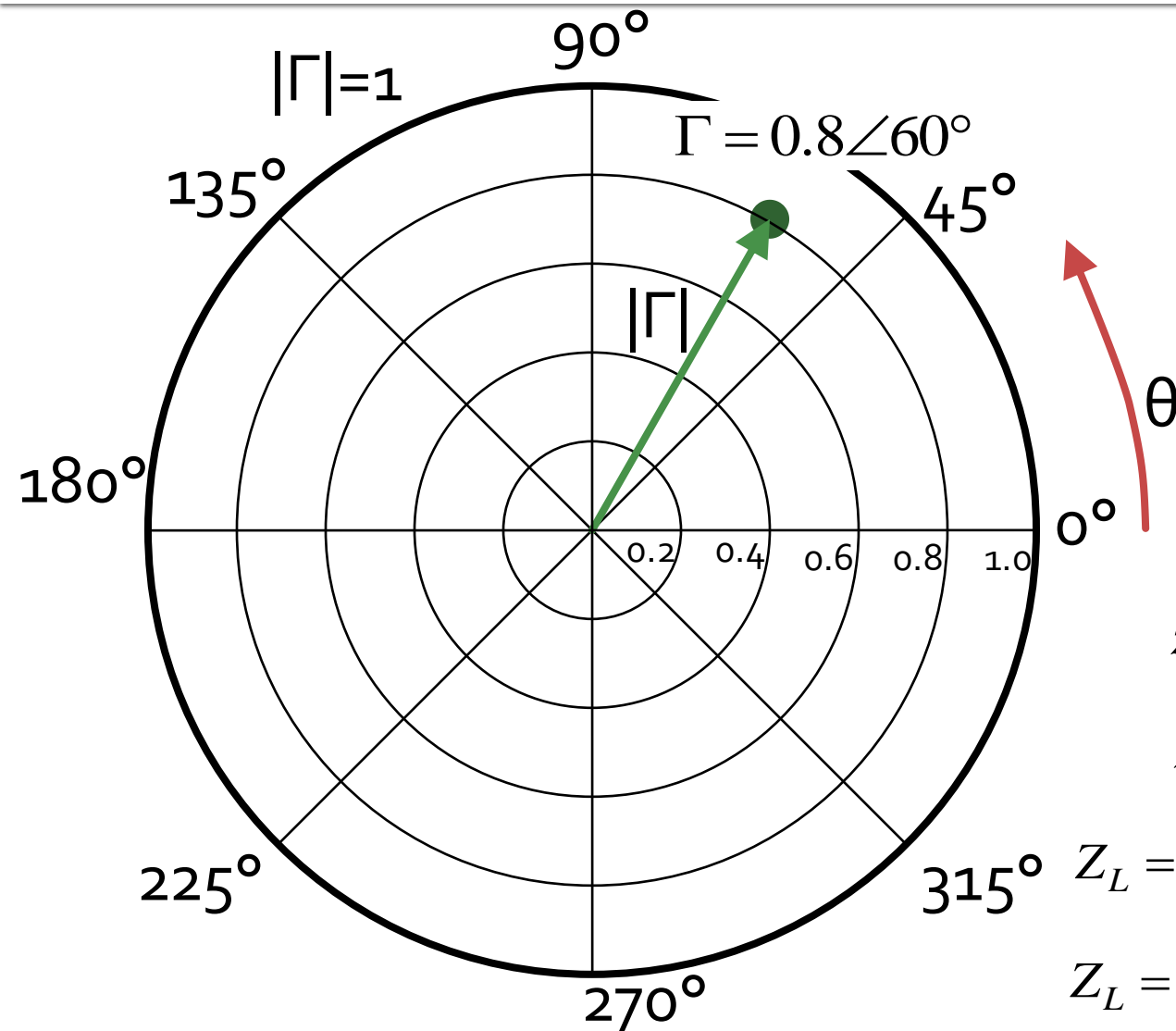
# The Smith Chart, reflection coefficient, Cartesian coordinate system



# The Smith Chart, reflection coefficient, Polar coordinate system



# The Smith Chart, reflection coefficient, impedance



$$\Gamma = |\Gamma| \cdot e^{j\theta}$$

$$\Gamma = |\Gamma| \angle \theta^\circ$$

$$\Gamma = 0.8 \angle 60^\circ$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1 + \Gamma}{1 - \Gamma} = \frac{1 + 0.8 \angle 60^\circ}{1 - 0.8 \angle 60^\circ}$$

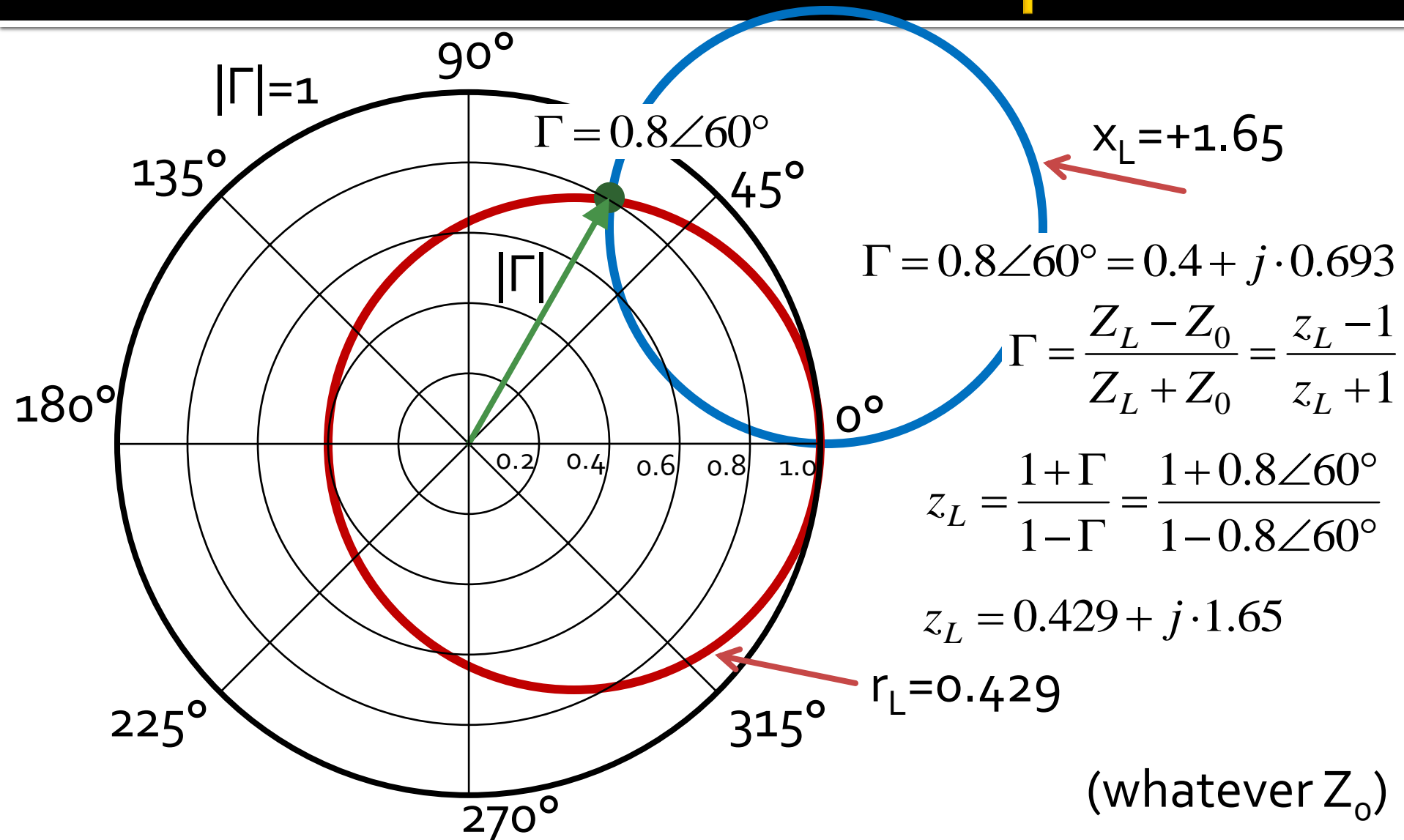
$$z_L = 0.429 + j \cdot 1.65$$

$$Z_L = Z_0 \cdot \frac{1 + \Gamma}{1 - \Gamma} = 50\Omega \cdot \frac{1 + 0.8 \angle 60^\circ}{1 - 0.8 \angle 60^\circ}$$

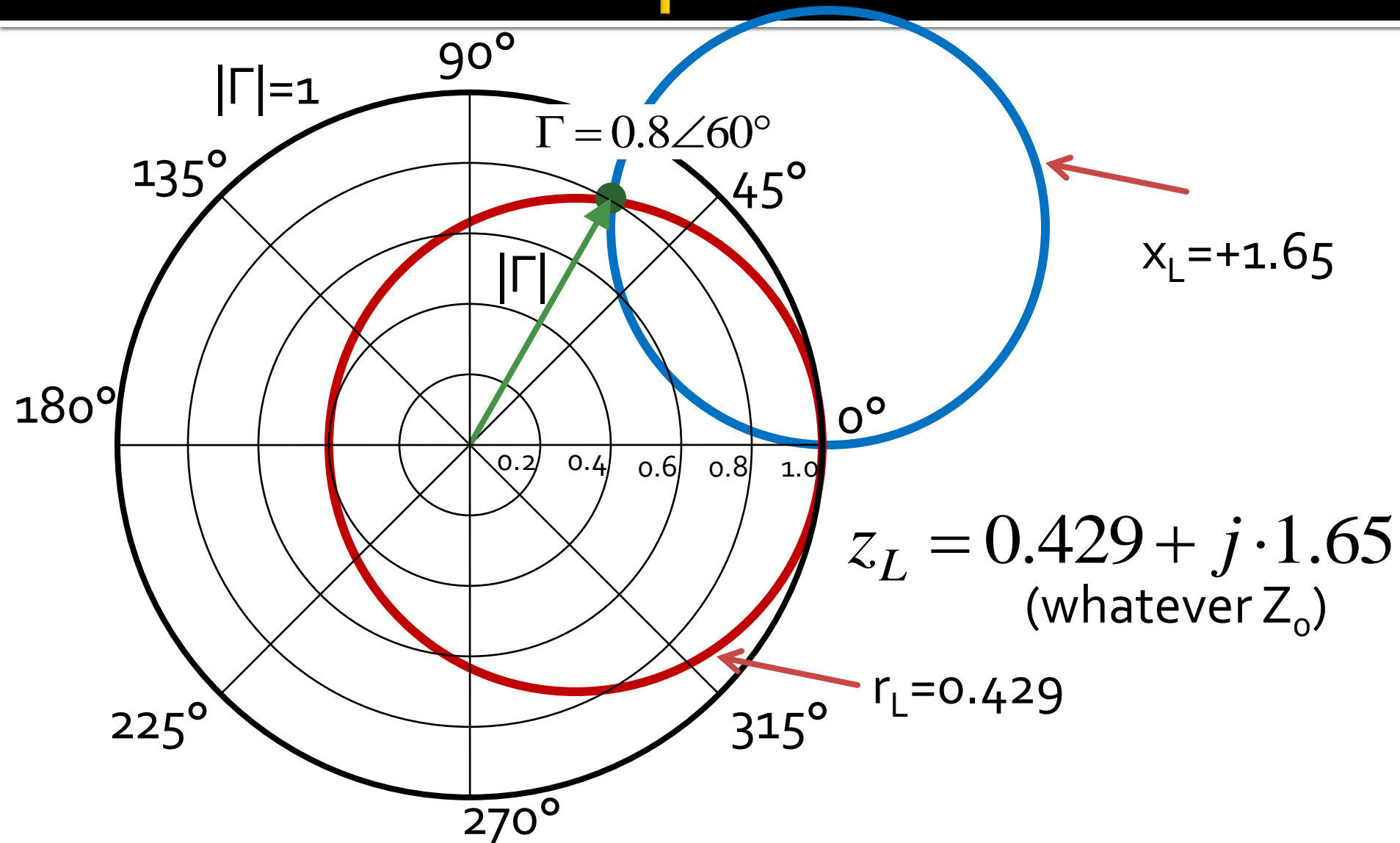
$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

# Equivalence

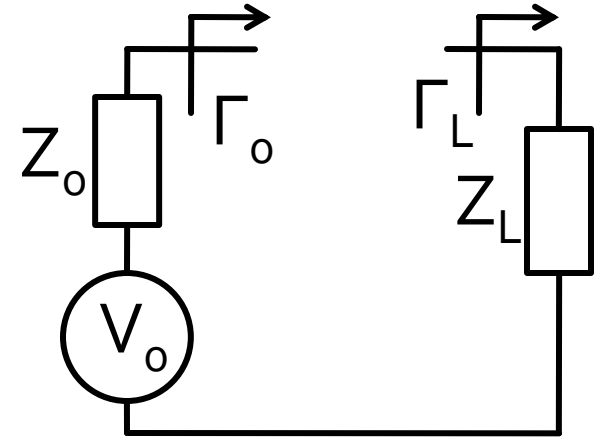
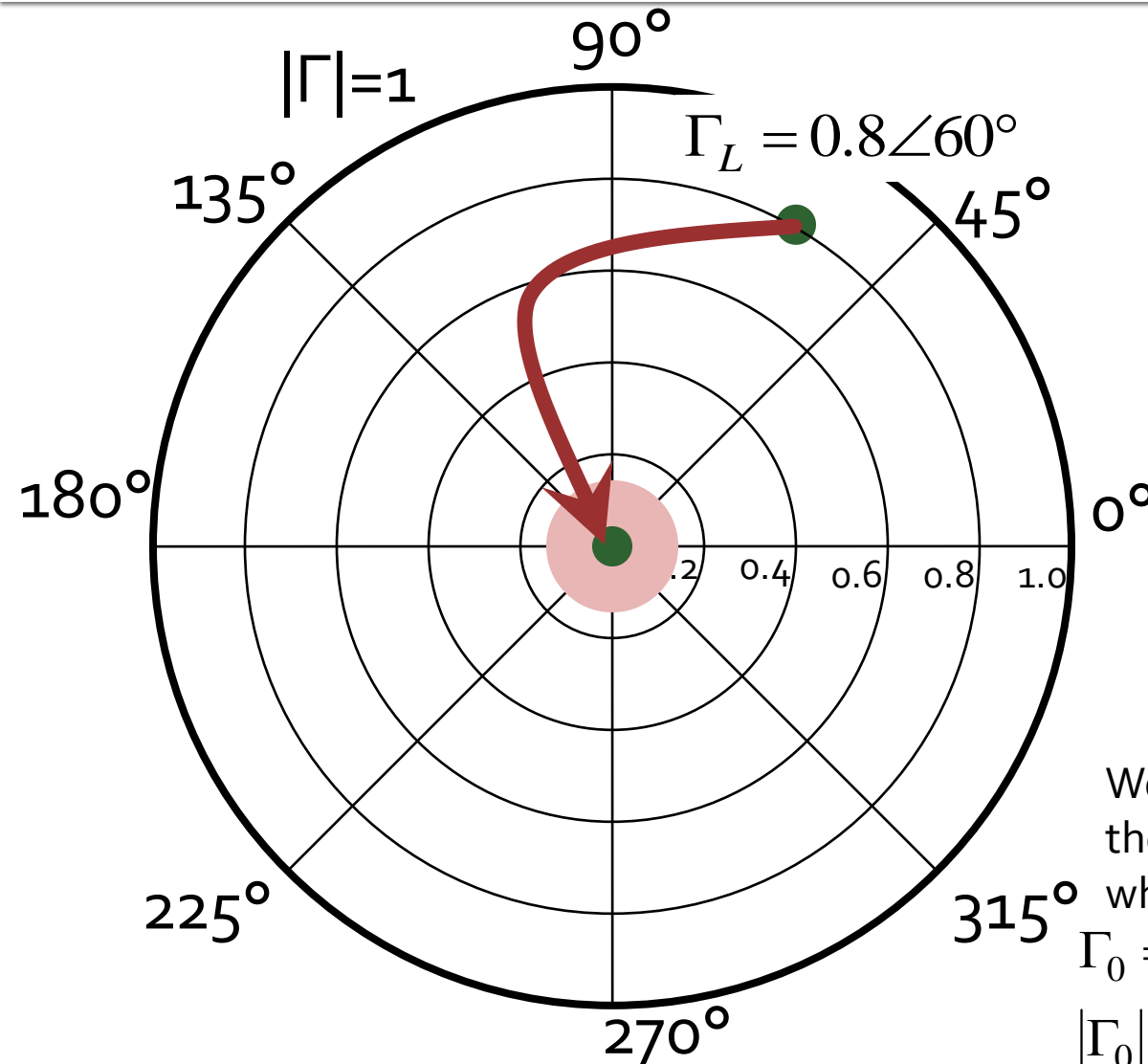
reflection coefficient  $\Leftrightarrow$  impedance



# The Smith Chart, reflection coefficient $\Leftrightarrow$ impedance



# The Smith Chart, reflection coefficient, matching



Matching  $Z_L$  load to  $Z_o$  source.  
We normalize  $Z_L$  over  $Z_o$

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

$$\Gamma_L = 0.8 \angle 60^\circ$$

We must move the point denoting the reflection coefficient in the area where with a  $Z_o$  source we have:

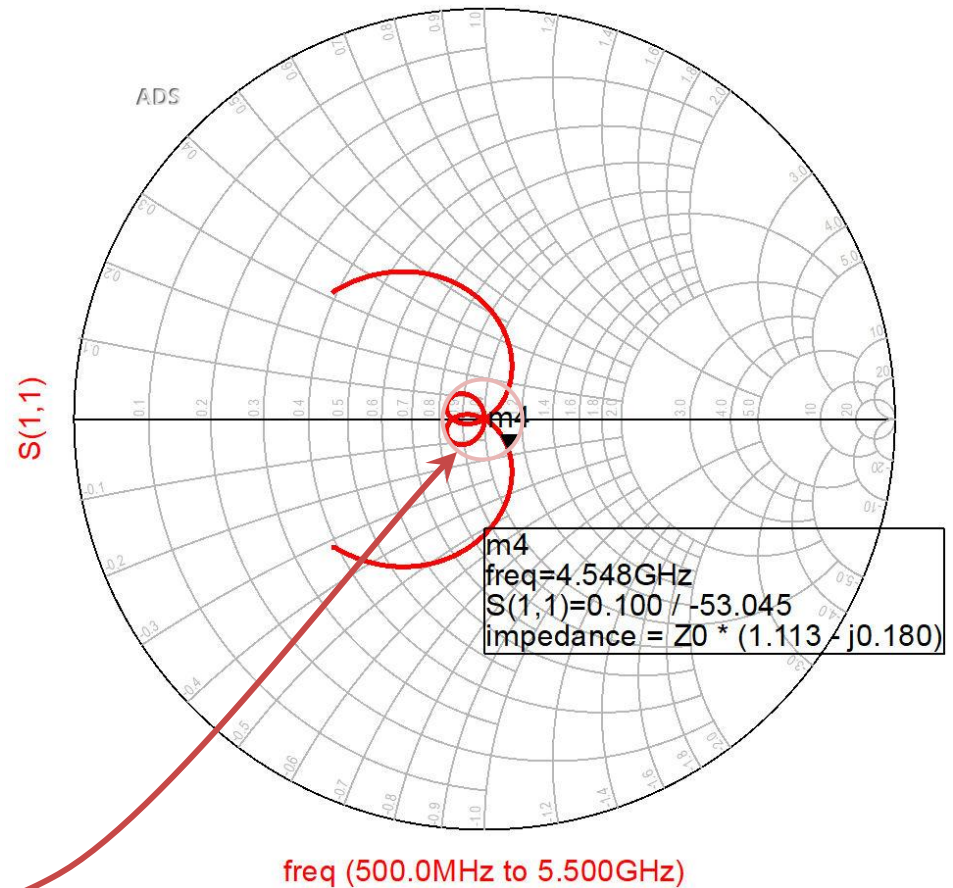
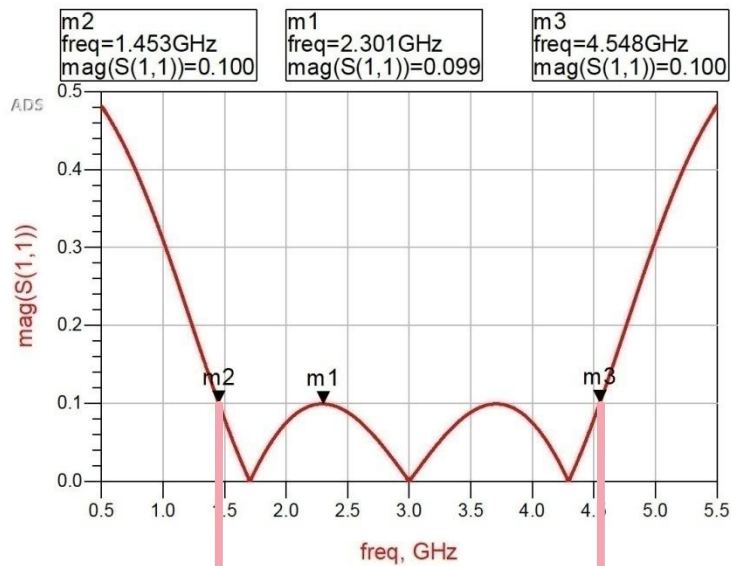
$$\Gamma_0 = 0 \text{ perfect match } \bullet$$

$$|\Gamma_0| \leq \Gamma_m \text{ "good enough" match } \circ$$



# Example

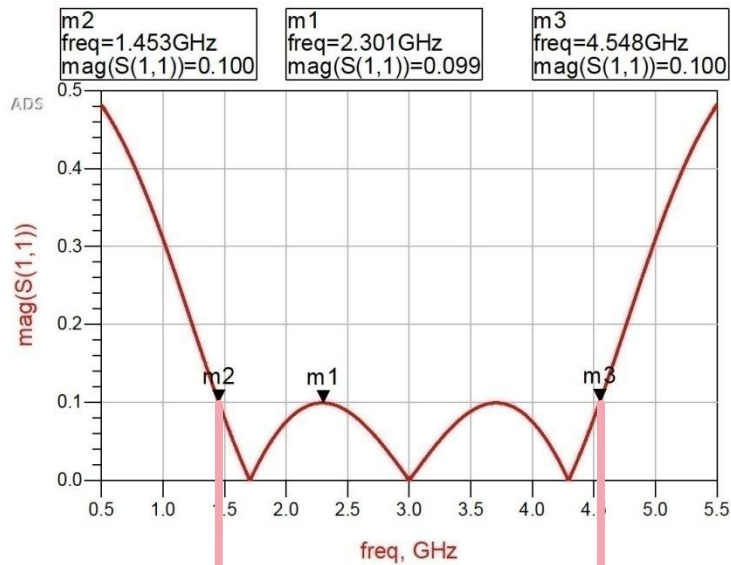
## ■ Laboratory 1



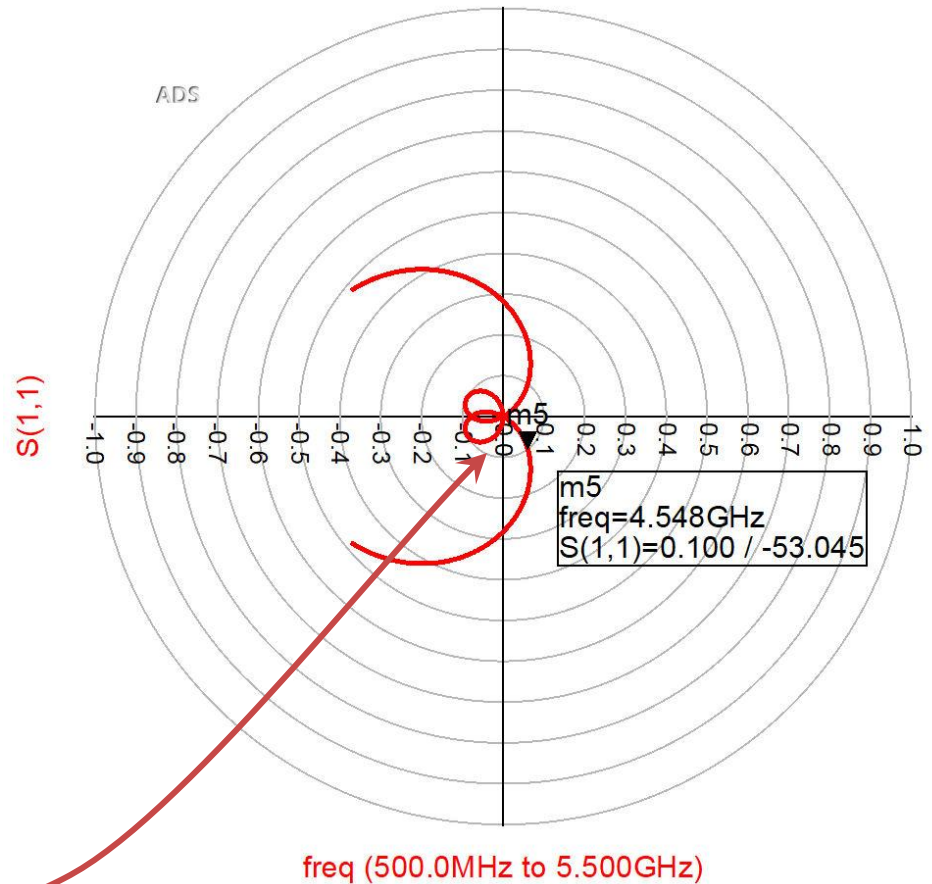
$$|\Gamma_0| \leq \Gamma_m$$

# Example

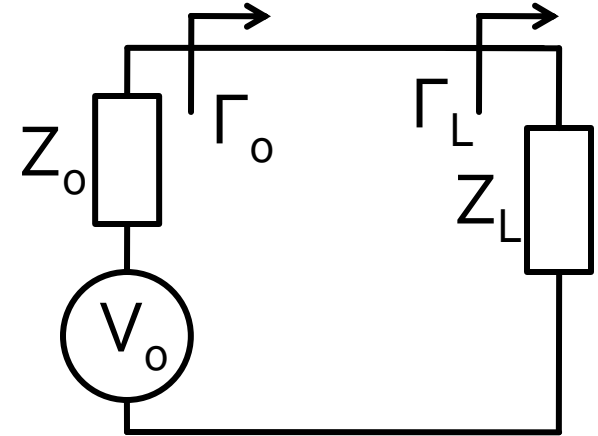
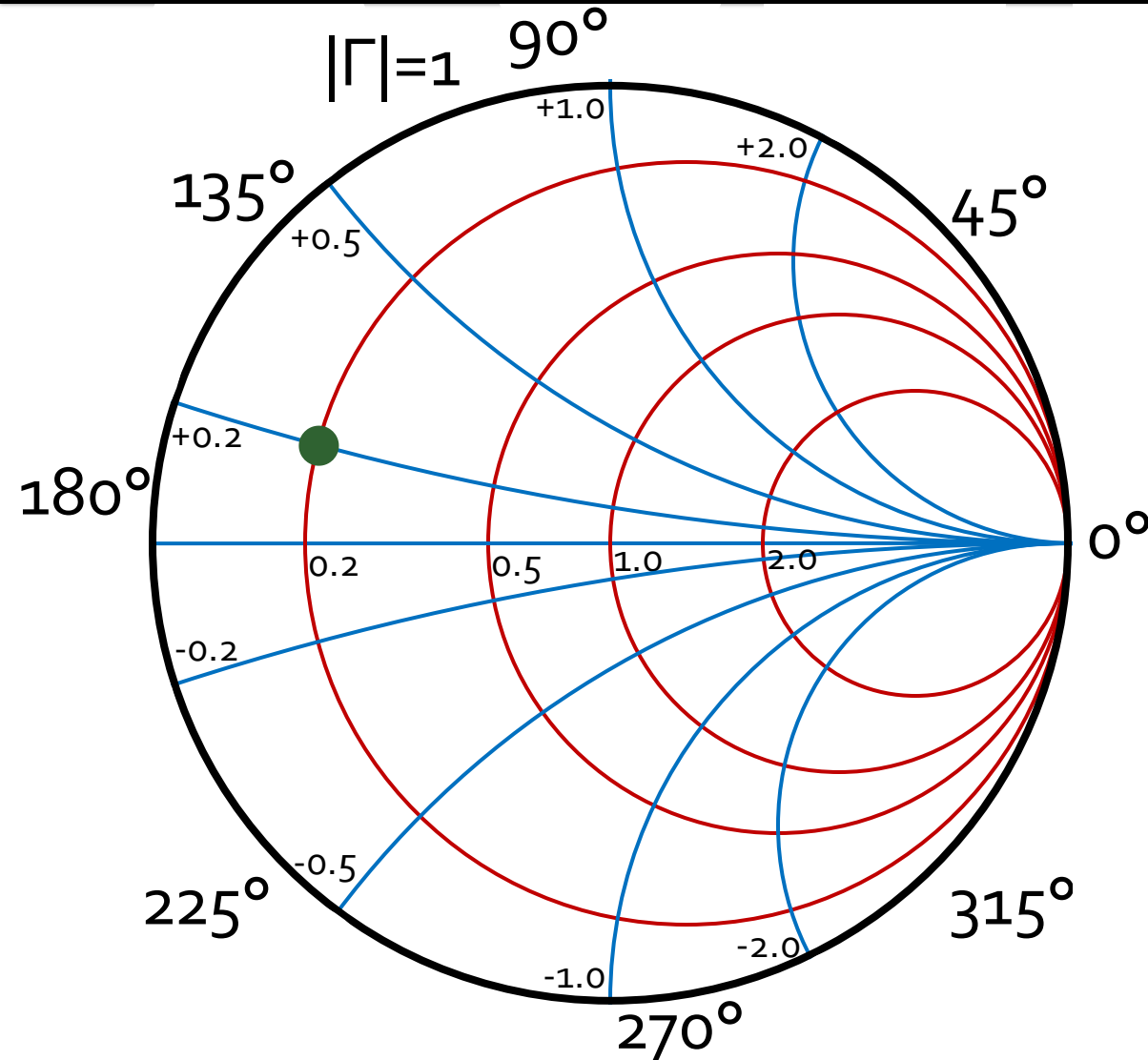
## ■ Laboratory 1



$$|\Gamma_0| \leq \Gamma_m$$



# The Smith Chart, impedance/reflection coefficient



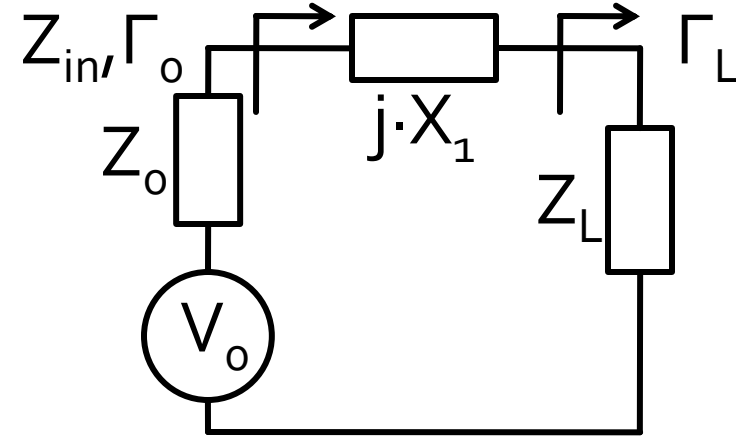
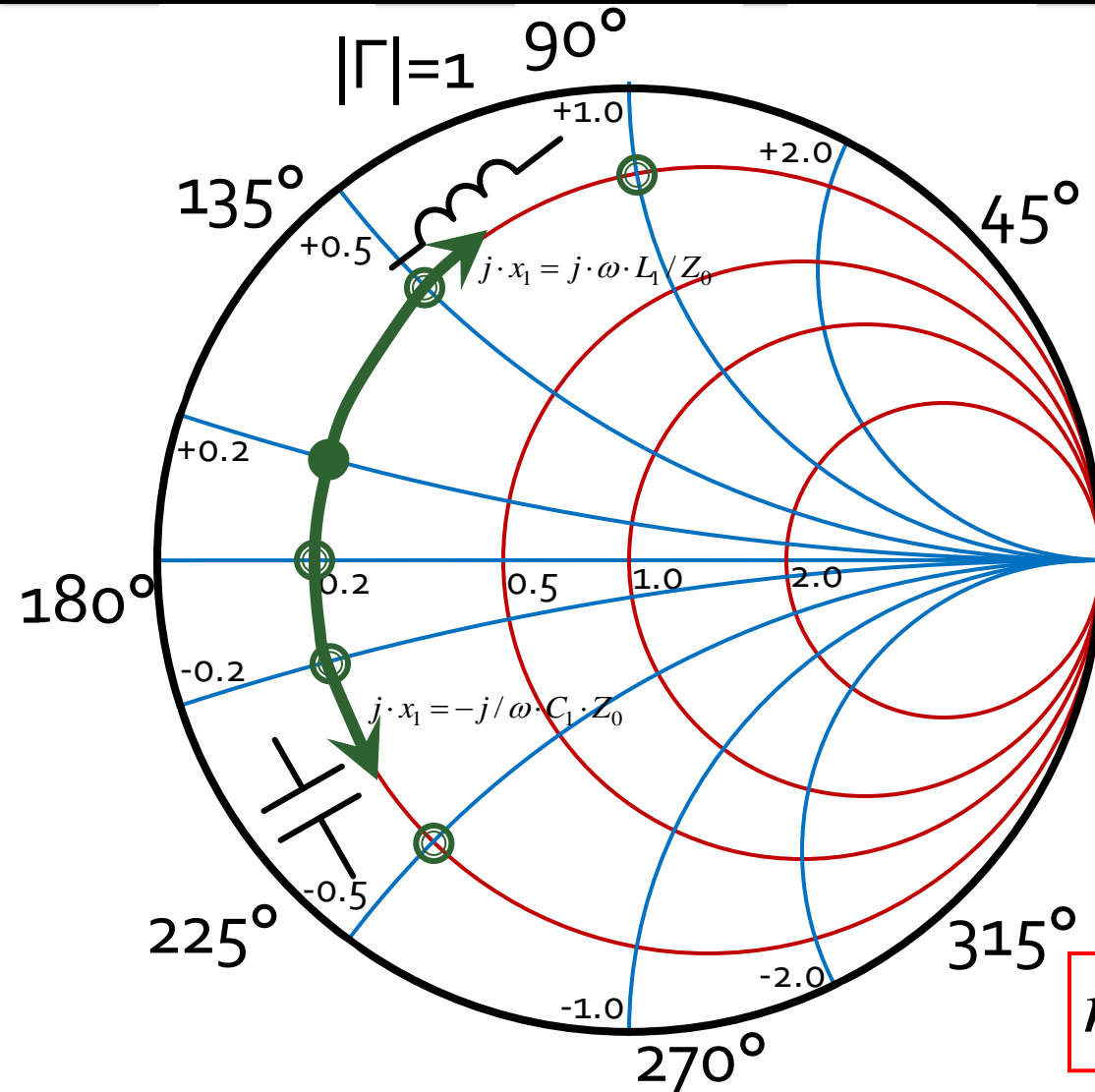
$$Z_0 = 50\Omega$$

$$Z_L = 10\Omega + j \cdot 10\Omega$$

$$z_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = \Gamma_0 = 0.678 \angle 156.5^\circ$$

# The Smith Chart, series reactance



$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10\Omega$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = 0.678 \angle 156.5^\circ$$

$$Z_{in} = Z_L + j \cdot X_1 = R_L + j \cdot (X_L + X_1)$$

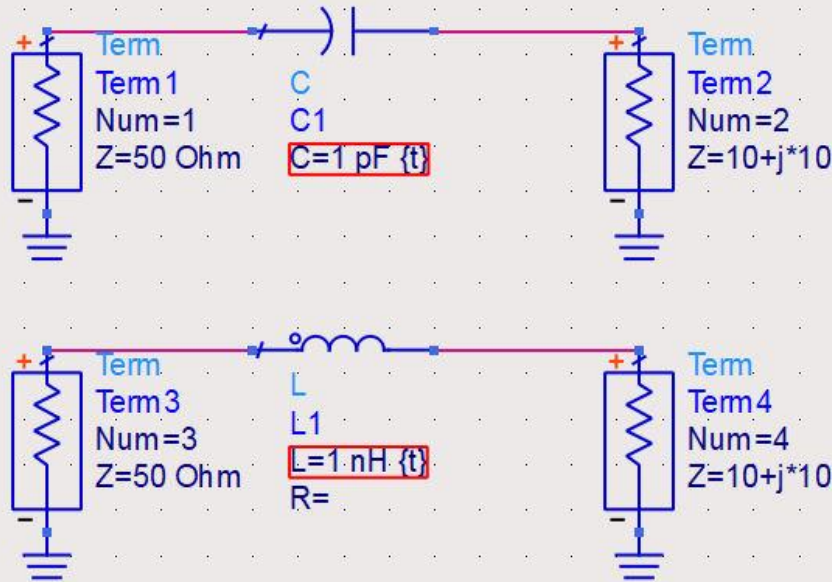
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

$$r_{in} = r_L$$

$$j \cdot x_1 = j \cdot \omega \cdot L_1 / Z_0 > 0$$

$$j \cdot x_1 = -j / \omega \cdot C_1 \cdot Z_0 < 0$$

# ADS, Smith Chart, series reactance



**Tune Parameters**

Simulate: While Slider Moves

Tune

Parameters: Include Opt Params, Enable/Disable..., Display Full Name

Snap Slider to Step

Traces and Values: Store..., Recall..., Trace Visibility..., Reset Values

Close Unassociated Data Displays

Update Schematic

Close Help

adaptaire\_LC\_lib:X\_S:schematic

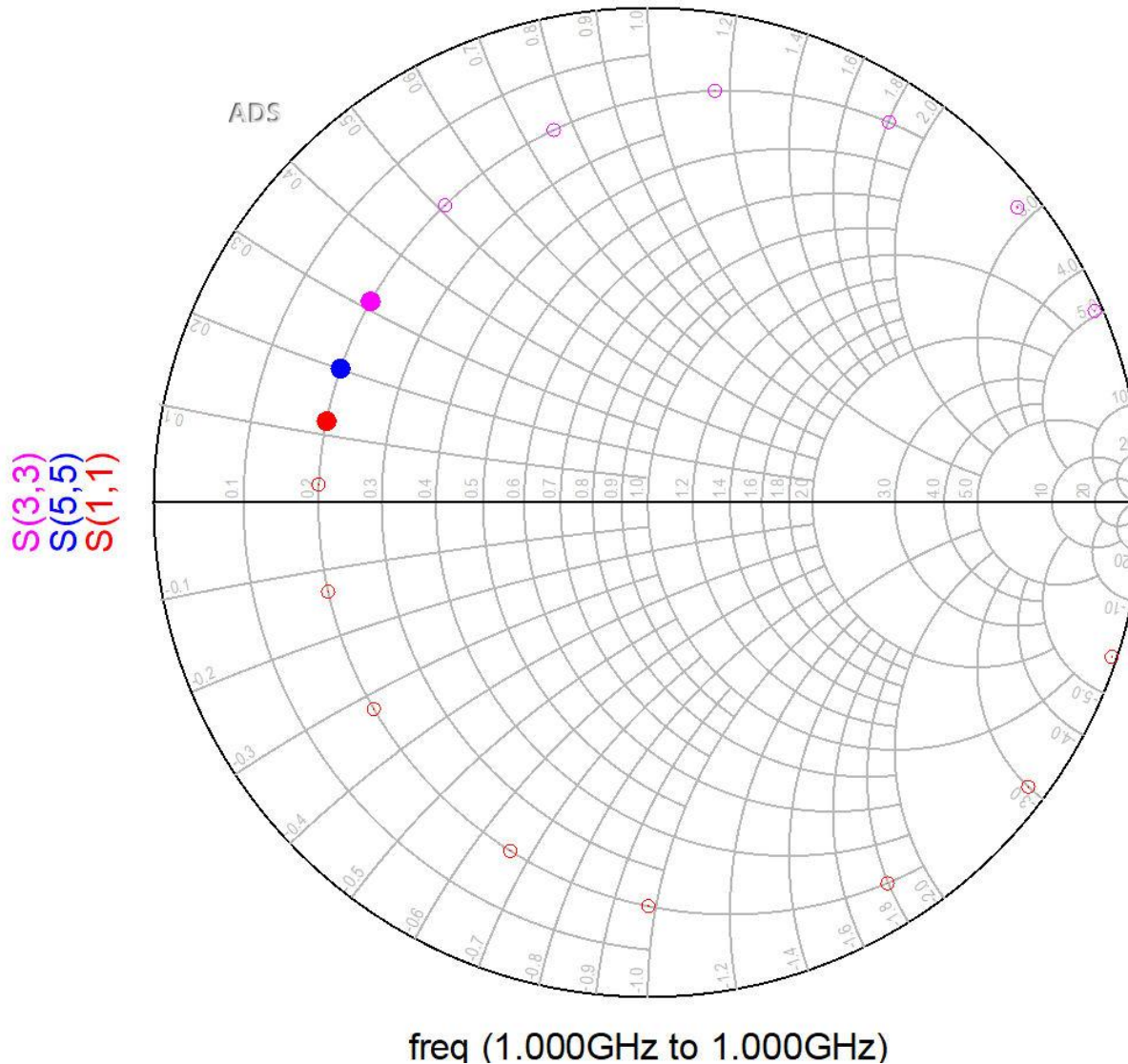
Component	Value	Max	Min	Step	Scale
C1.C (pF)	39.605	50	0.5	0.1	Lin
L1.L (nH)	0.895	40	0.5	0.1	Lin



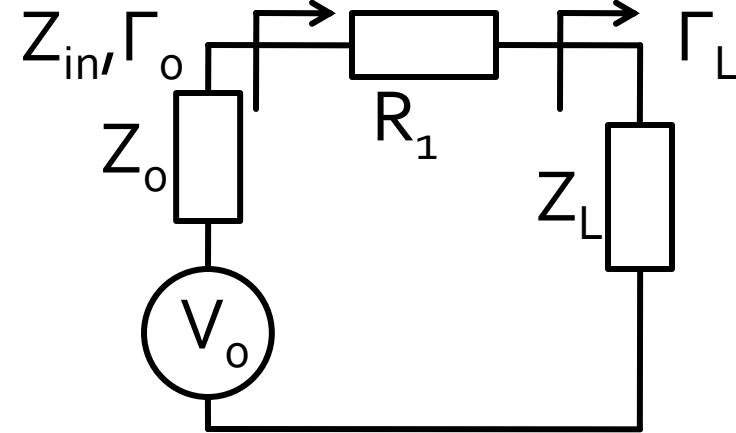
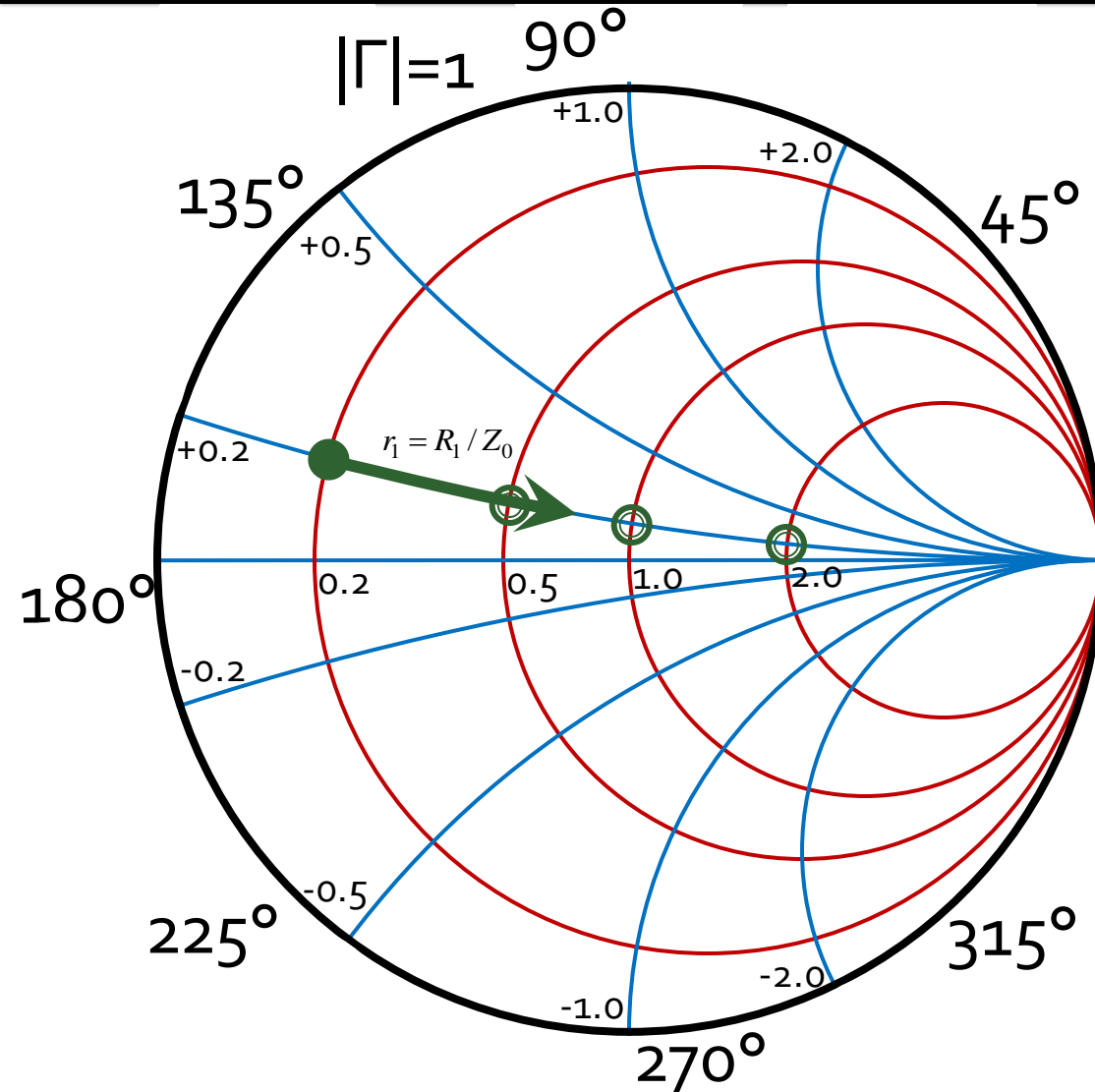
S-PARAMETERS

S\_Param  
SP1  
Freq=1.0 GHz

# ADS, Smith Chart, series reactance



# The Smith Chart, series resistance



$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10\Omega$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

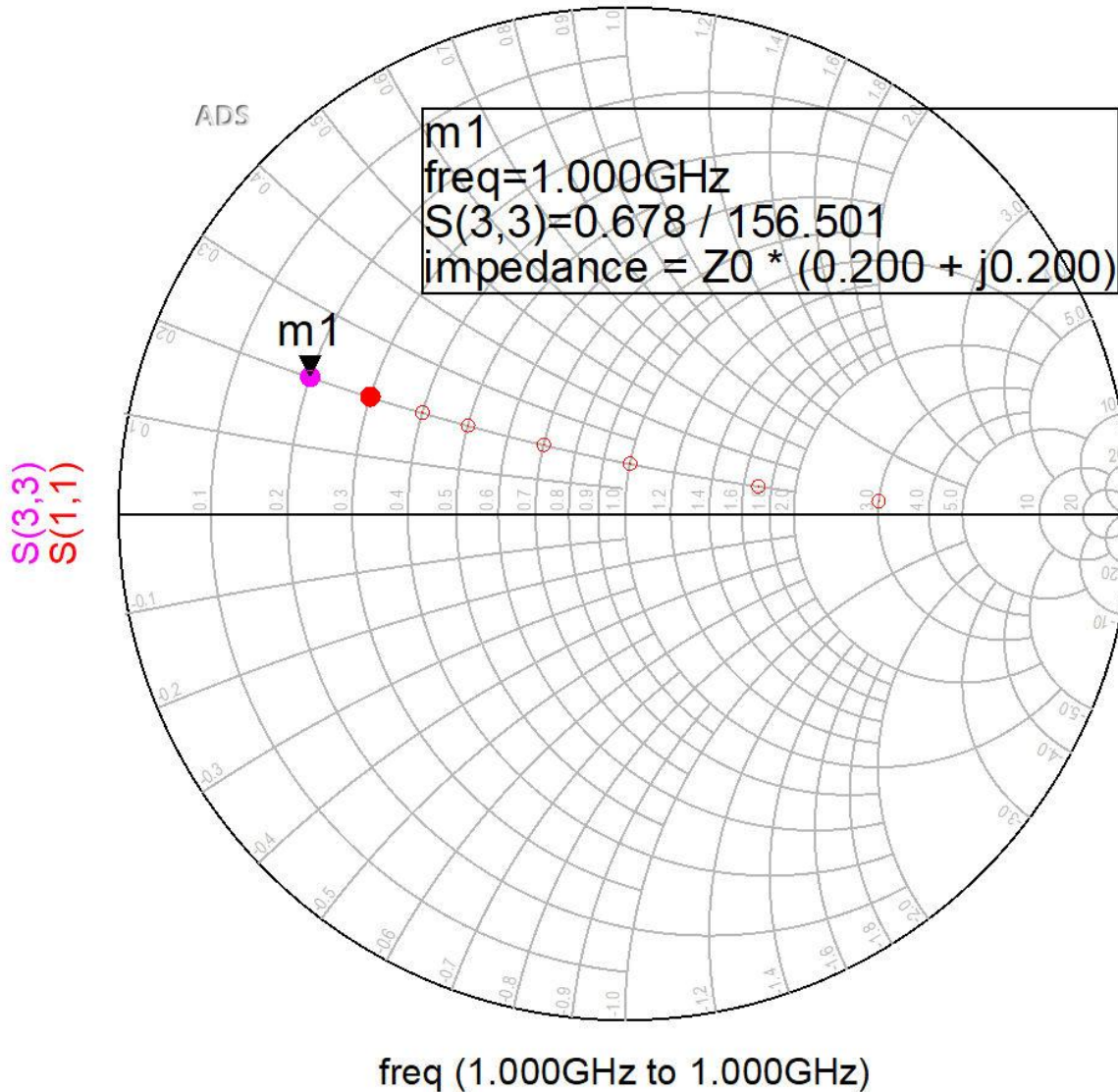
$$\Gamma_L = 0.678 \angle 156.5^\circ$$

$$Z_{in} = Z_L + R_1 = (R_L + R_1) + j \cdot X_L$$

$$z_{in} = z_L + r_1 = (r_L + r_1) + j \cdot x_L$$

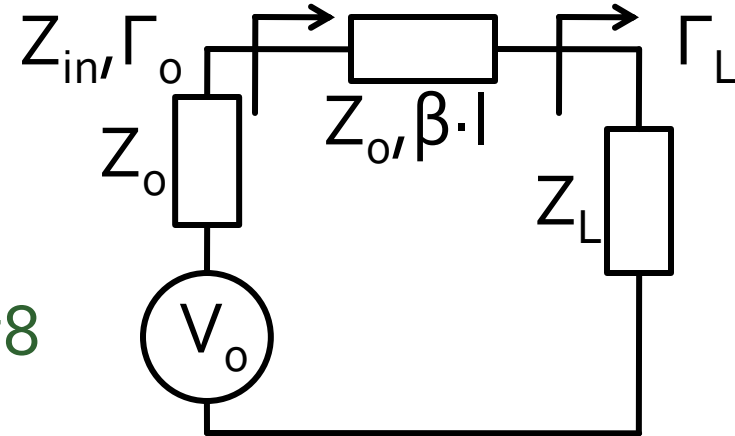
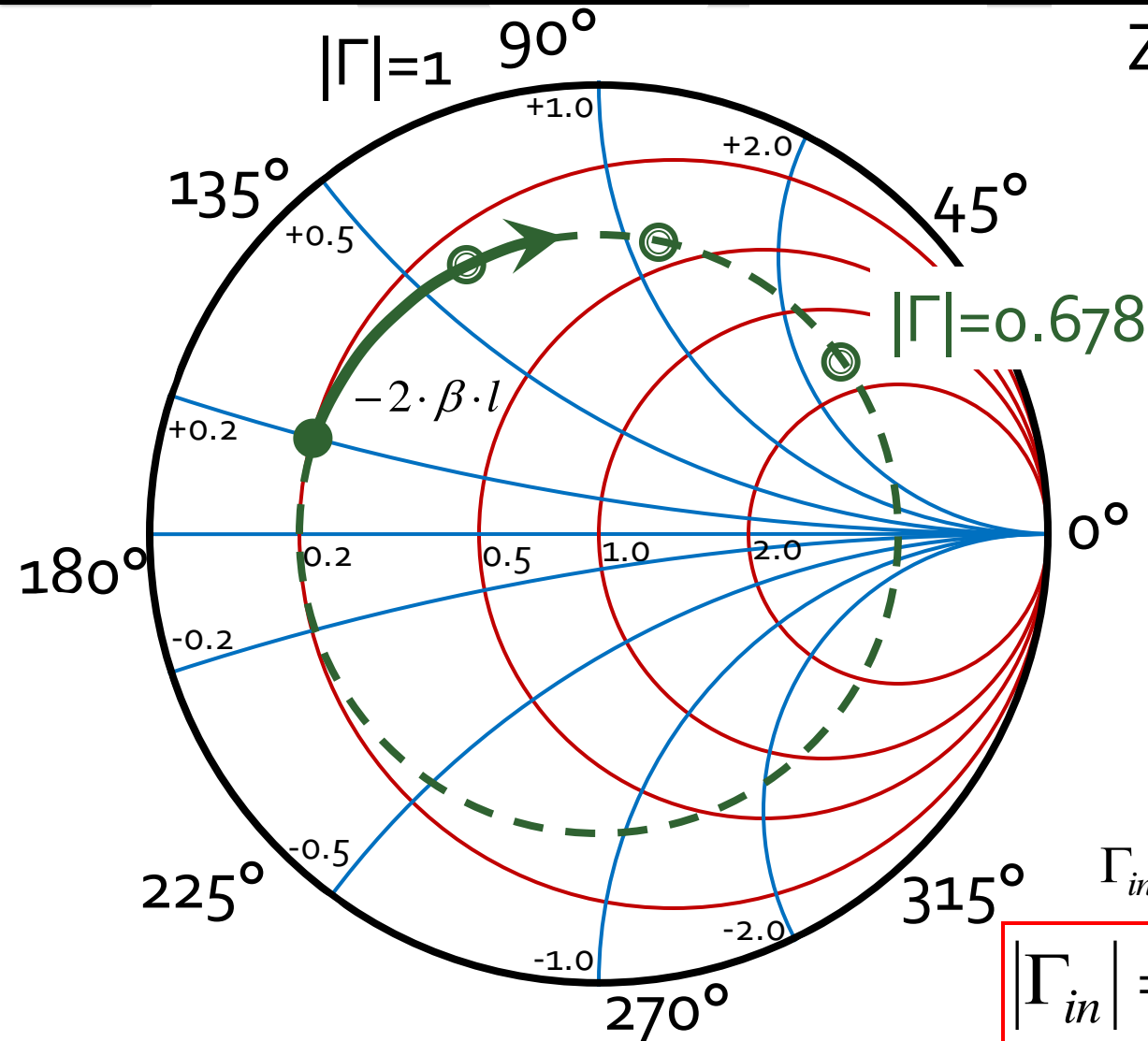
$$x_{in} = x_L \quad r_{in} = r_L + R_1 / Z_0$$

# ADS, Smith Chart, series resistance





# The Smith Chart, series transmission line, $Z_0$



$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10\Omega$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

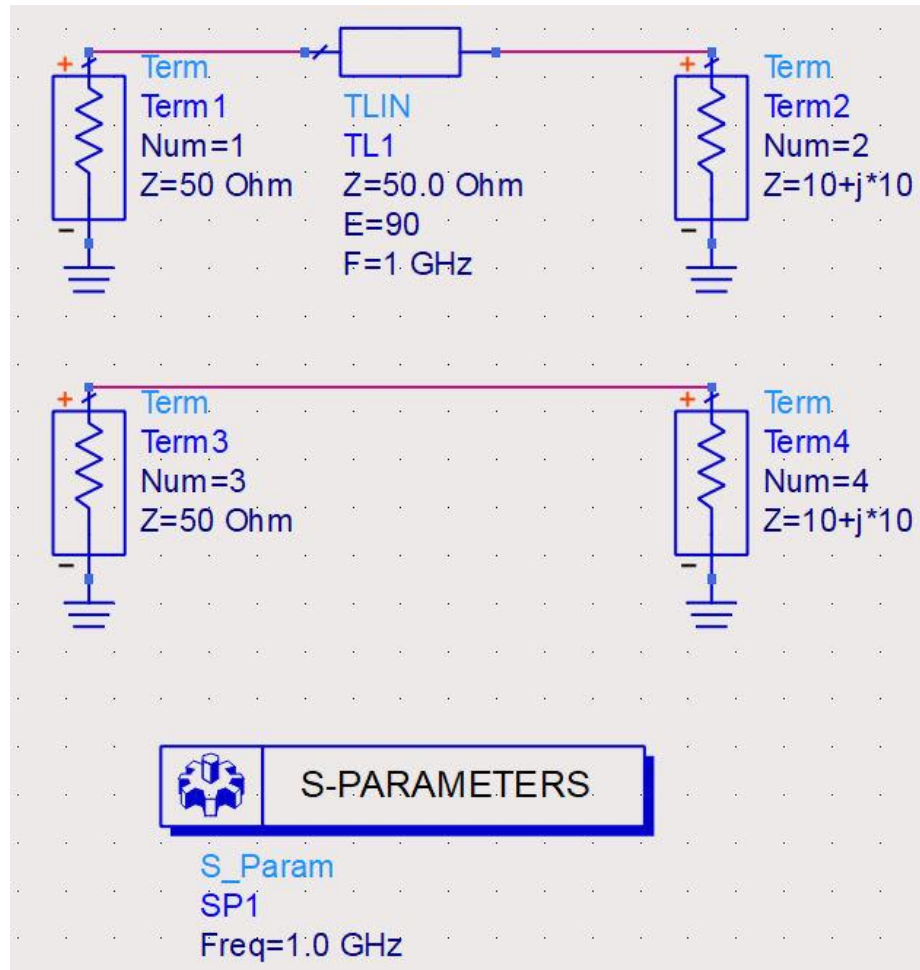
$$\Gamma_L = 0.678 \angle 156.5^\circ$$

$$Z_{in} = Z_0 \cdot \frac{1 + \Gamma_L \cdot e^{-2j \cdot \beta \cdot l}}{1 - \Gamma_L \cdot e^{-2j \cdot \beta \cdot l}}$$

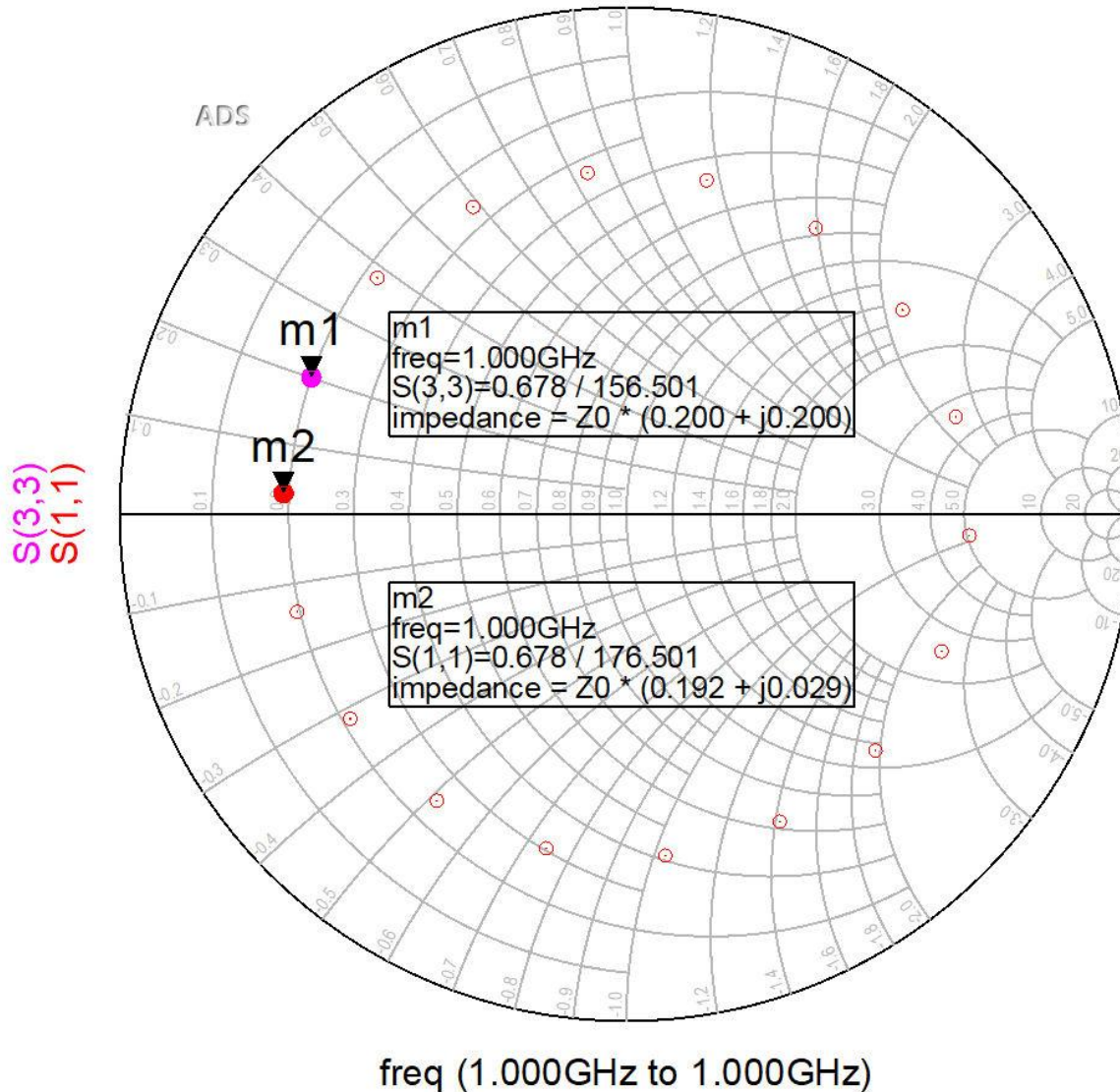
$$\Gamma_{in} = \Gamma_L \cdot e^{-2j \cdot \beta \cdot l}$$

$$|\Gamma_{in}| = |\Gamma_L| \quad \arg(\Gamma_{in}) = \arg(\Gamma_L) - 2 \cdot \beta \cdot l$$

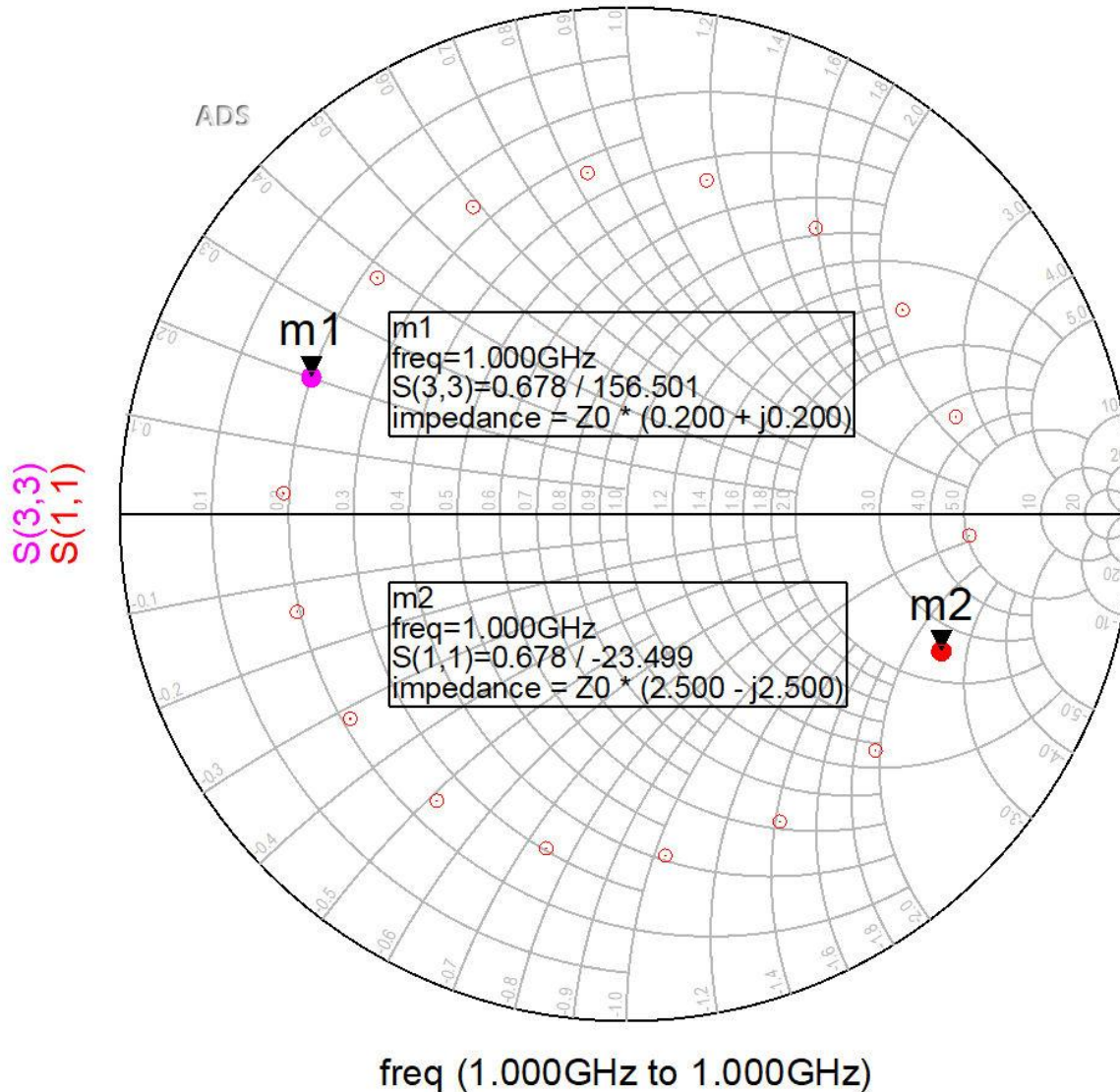
# ADS, Smith Chart, series transmission line



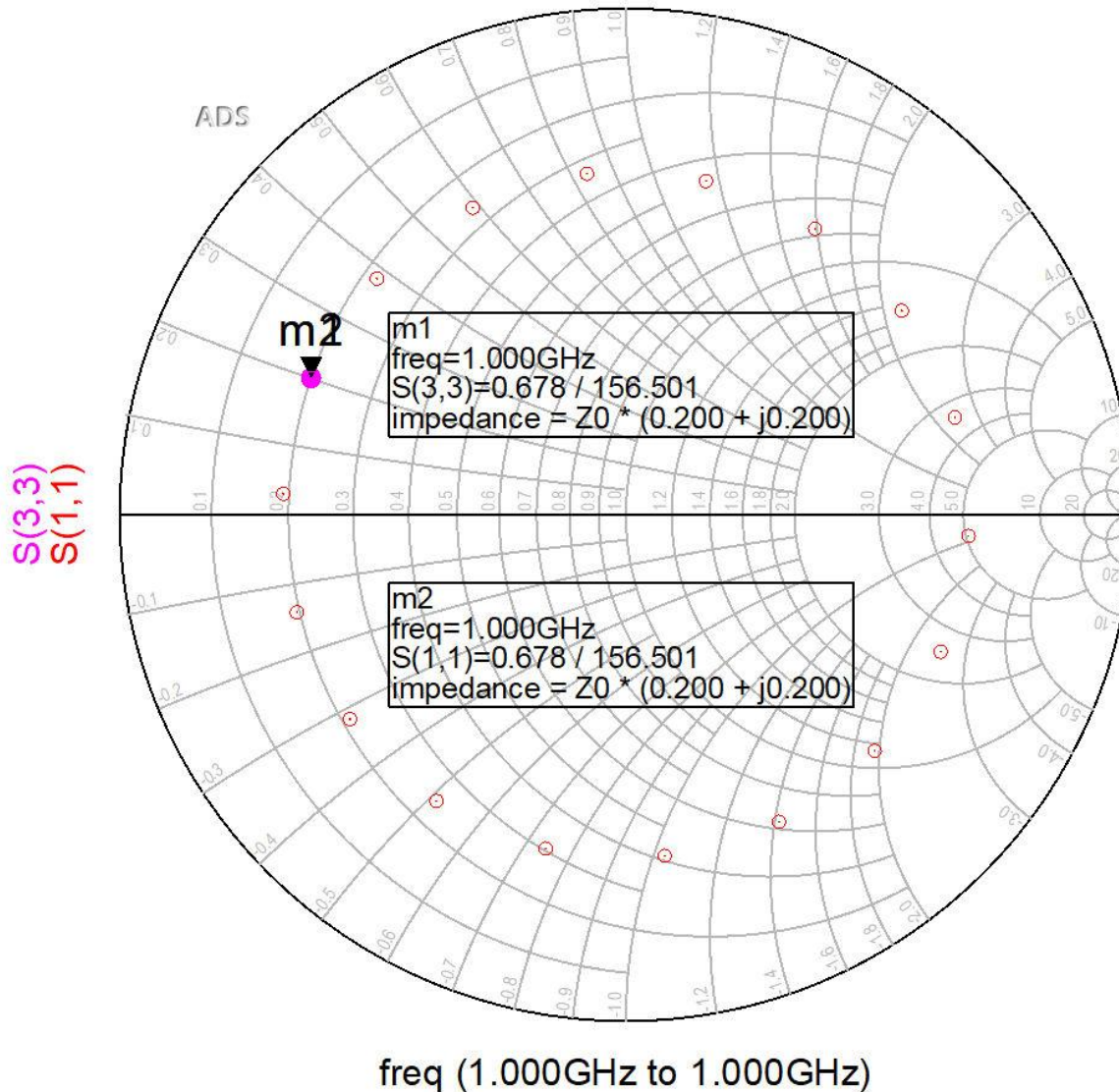
# ADS, Smith Chart, series transmission line



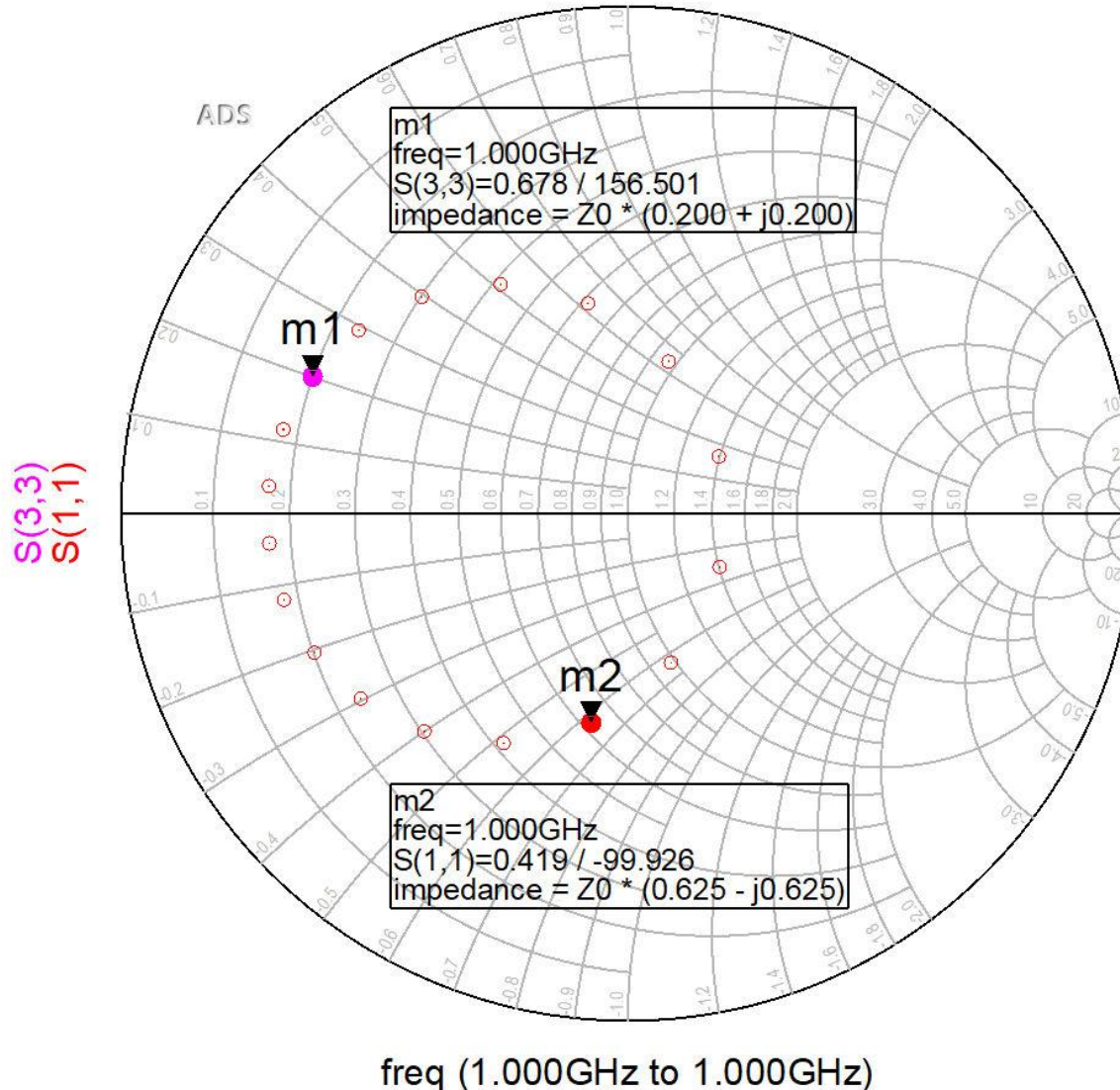
# ADS, Smith Chart, series transmission line, $E=90^\circ$



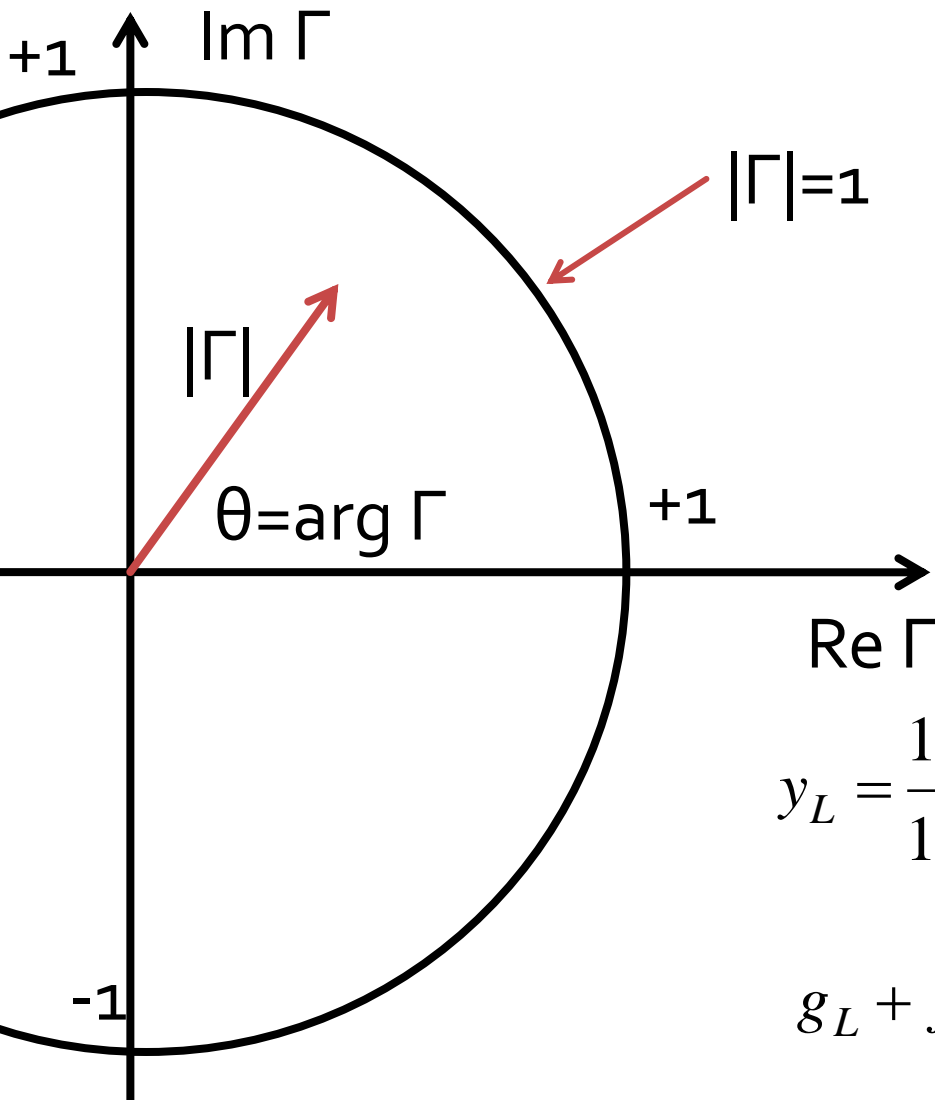
# ADS, Smith Chart, series transmission line, $E=180^\circ$



# ADS, Smith Chart, series transmission line, $Z=25\Omega \neq Z_0$



# The Admittance Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} = |\Gamma| \cdot e^{j\theta}$$

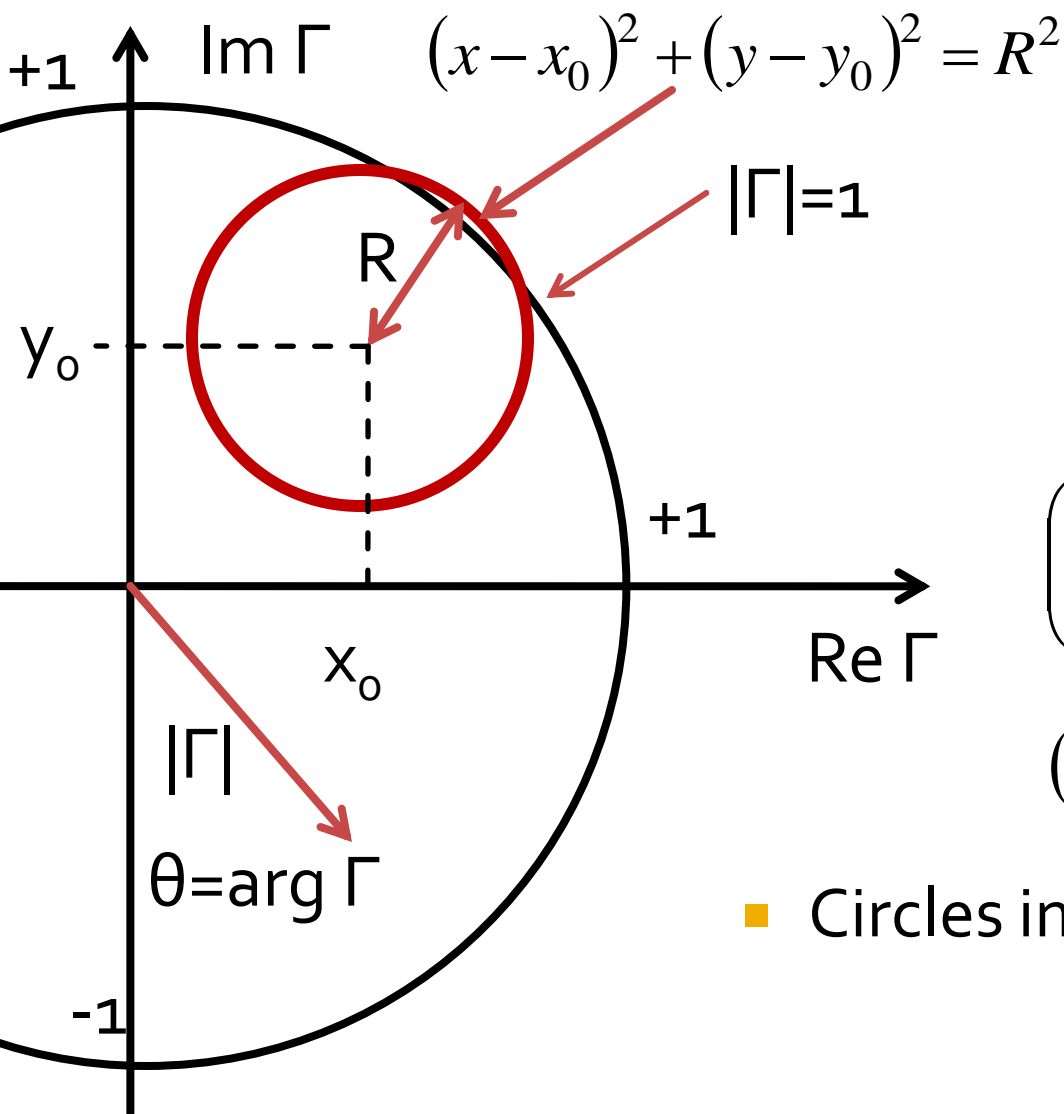
$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$z_L = \frac{1 + |\Gamma| \cdot e^{j\theta}}{1 - |\Gamma| \cdot e^{j\theta}} = r_L + j \cdot x_L$$

$$y_L = \frac{1 - |\Gamma| \cdot e^{j\theta}}{1 + |\Gamma| \cdot e^{j\theta}} = \frac{1}{r_L + j \cdot x_L} = g_L + j \cdot b_L$$

$$g_L + j \cdot b_L = \frac{(1 - \Gamma_r) - j \cdot \Gamma_i}{(1 + \Gamma_r) + j \cdot \Gamma_i}$$

# The Admittance Smith Chart



$$g_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 + \Gamma_r)^2 + \Gamma_i^2}$$

$$b_L = \frac{-2 \cdot \Gamma_i}{(1 + \Gamma_r)^2 + \Gamma_i^2}$$

## ■ Rearranged

$$\left( \Gamma_r + \frac{g_L}{1 + g_L} \right)^2 + \Gamma_i^2 = \left( \frac{1}{1 + g_L} \right)^2$$

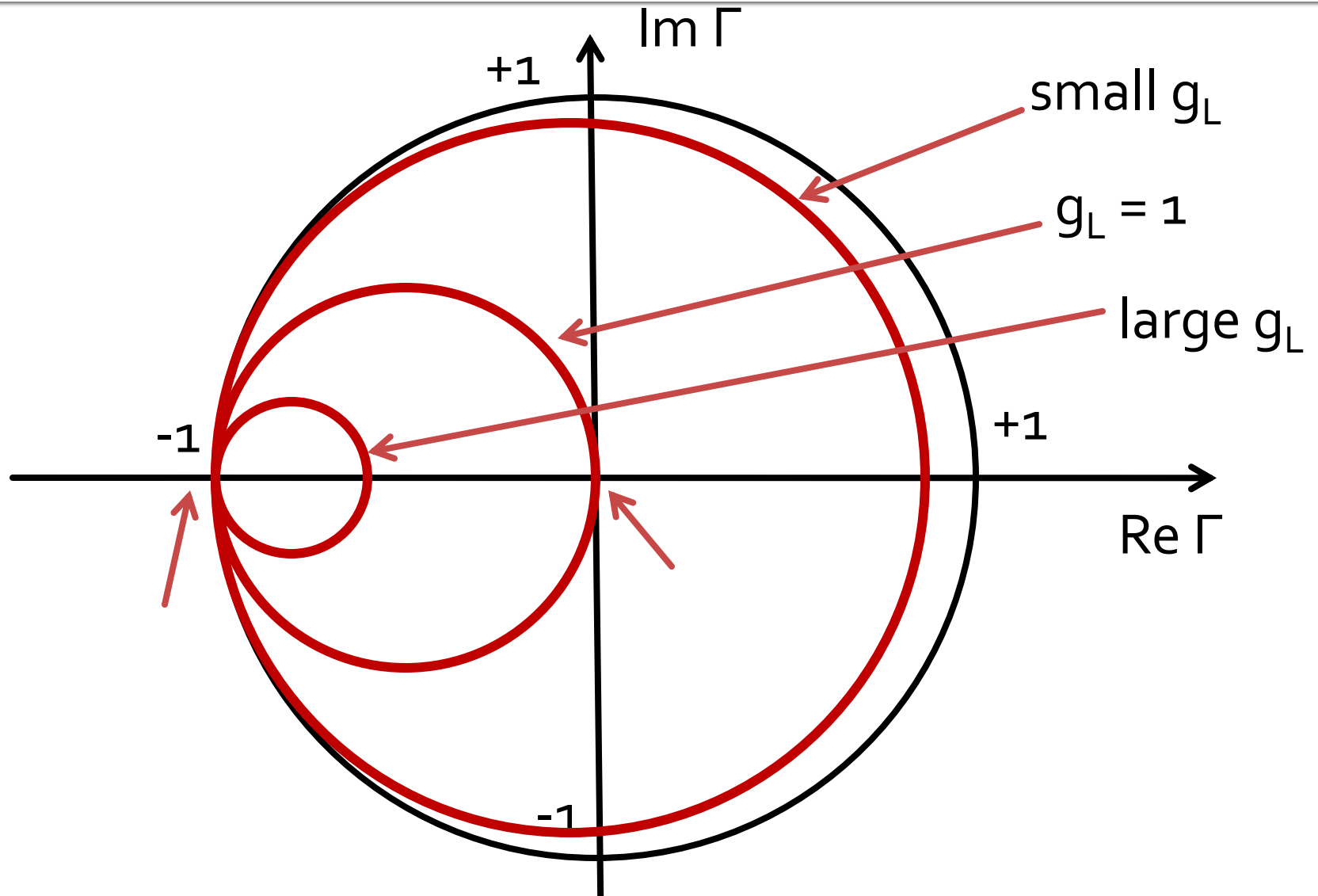
$$(\Gamma_r + 1)^2 + \left( \Gamma_i + \frac{1}{b_L} \right)^2 = \left( \frac{1}{b_L} \right)^2$$

- Circles in the  $(\Gamma_r, \Gamma_i)$  complex plane

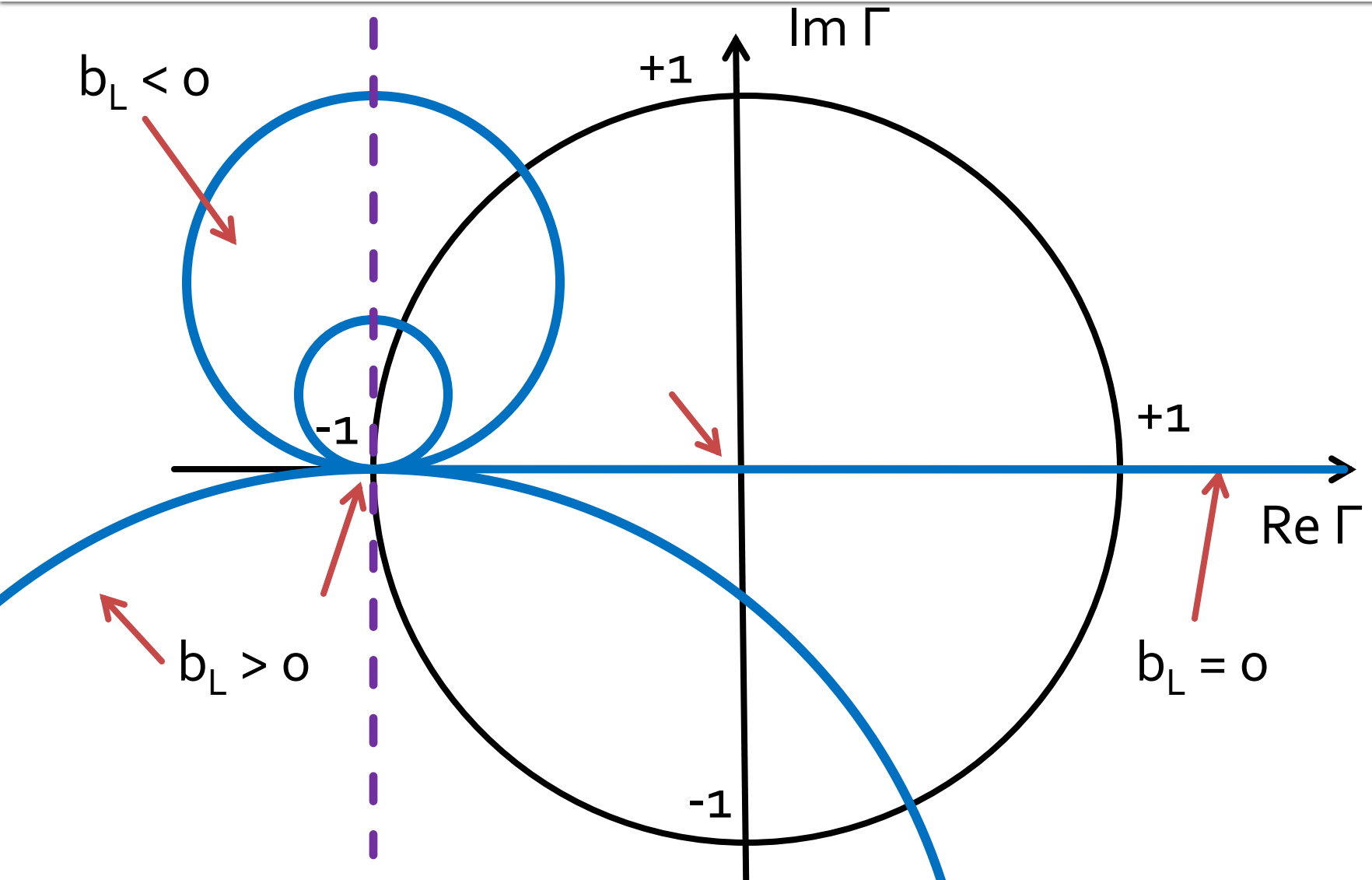
$$(x - x_0)^2 + (y - y_0)^2 = R^2$$



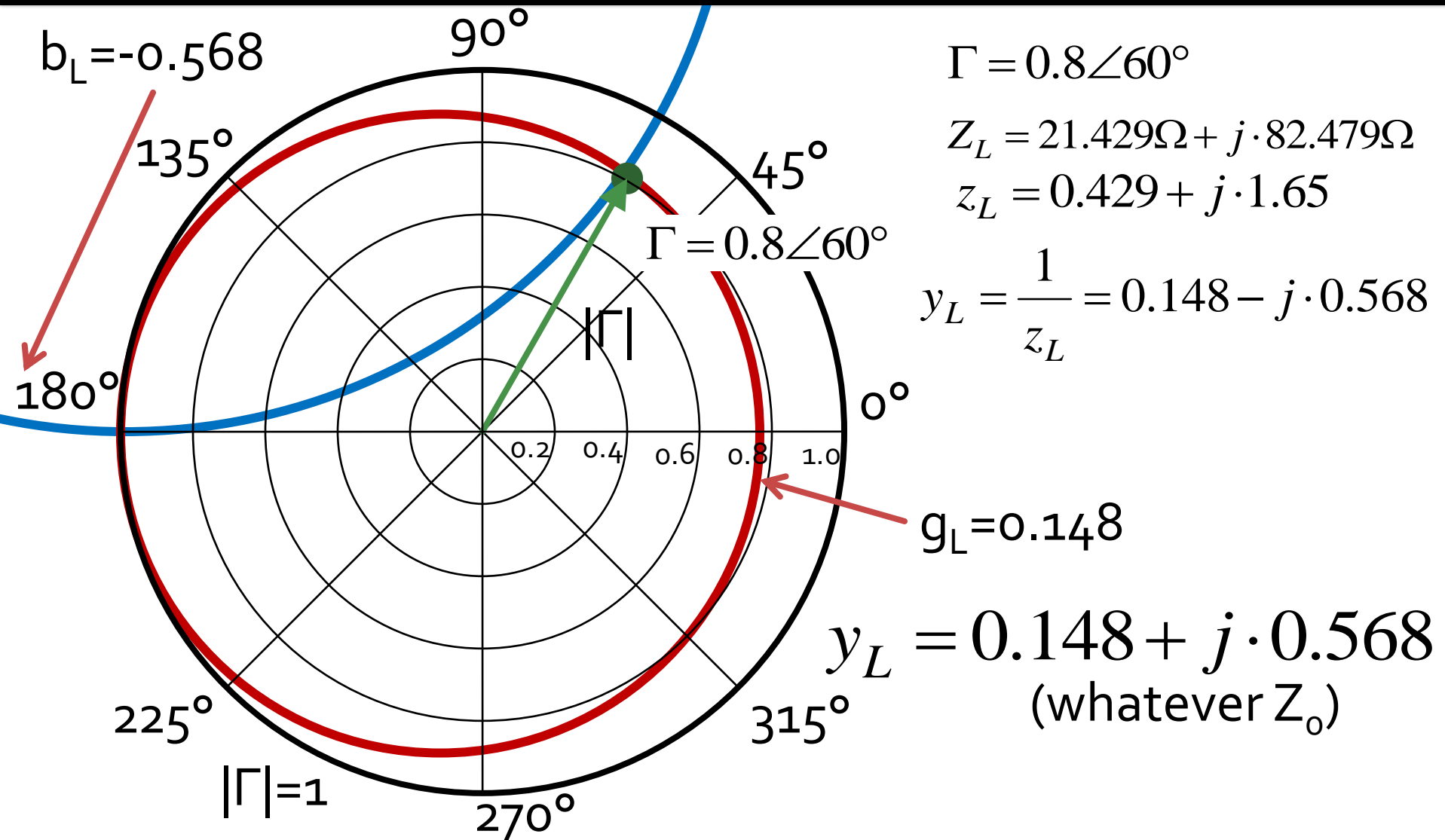
# The Smith Chart, conductance



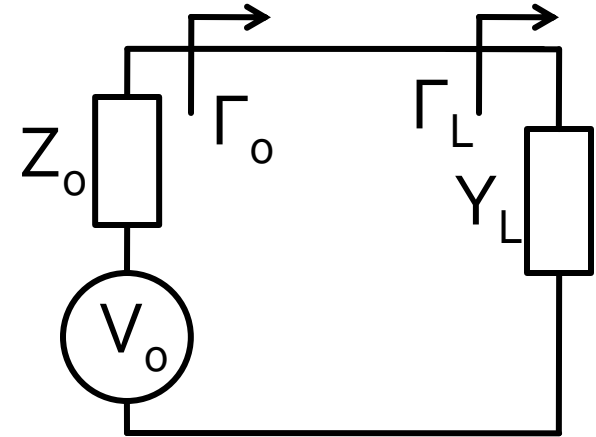
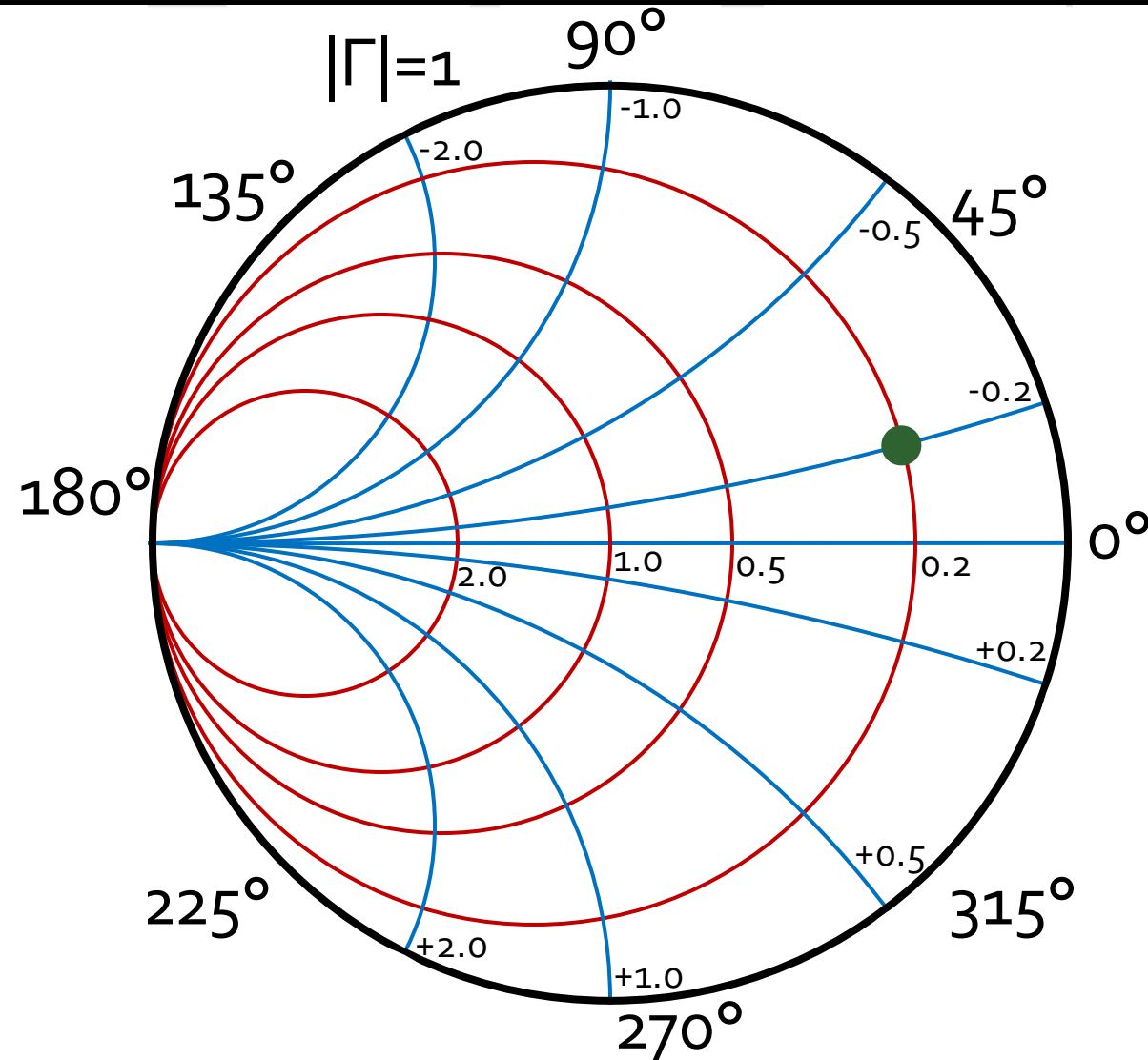
# The Smith Chart, susceptance



# The Smith Chart, reflection coefficient $\Leftrightarrow$ admittance



# The Smith Chart, reflection coefficient $\Leftrightarrow$ admittance



$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$Z_L = 125\Omega + j \cdot 125\Omega$$

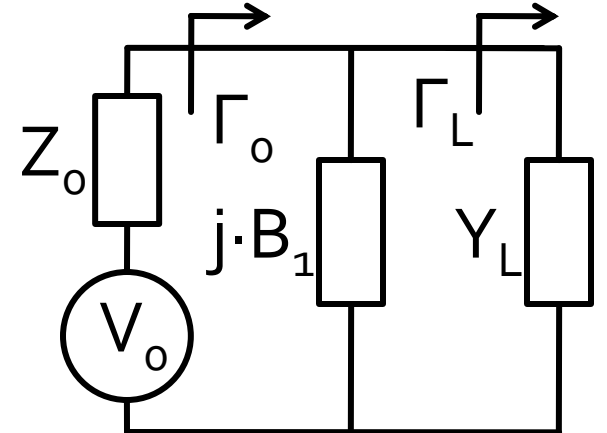
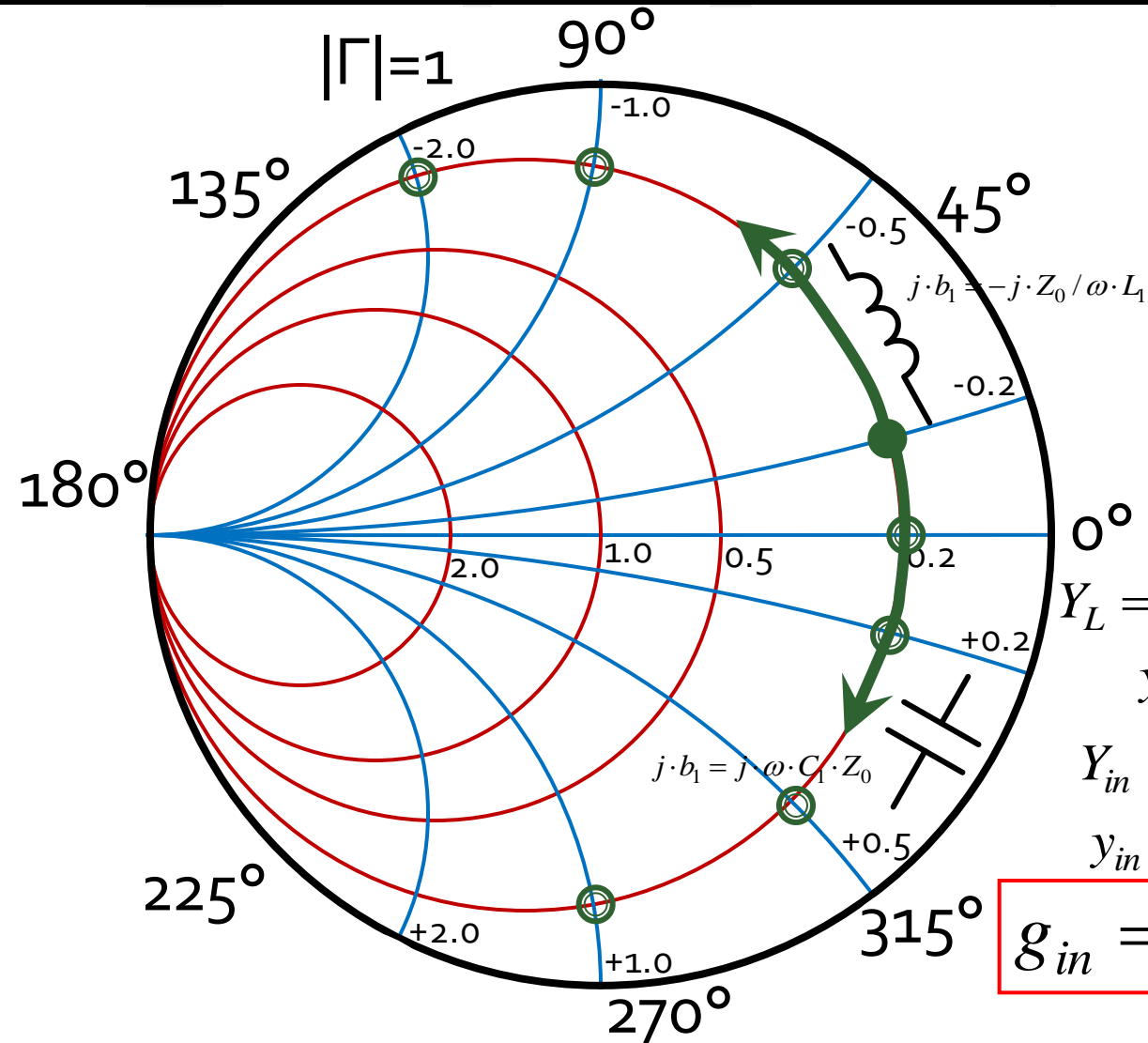
$$z_L = 2.5 + j \cdot 2.5$$

$$\Gamma_L = \Gamma_0 = 0.678 \angle 23.5^\circ$$

$$Y_L = \frac{1}{Z_L} = 0.004S - j \cdot 0.004S$$

$$y_L = \frac{1}{z_L} = \frac{Y_L}{Y_0} = 0.2 - j \cdot 0.2$$

# The Smith Chart, shunt susceptance



$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678 \angle 23.5^\circ$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

$$Y_{in} = Y_L + j \cdot B_1 = G_L + j \cdot (B_L + B_1)$$

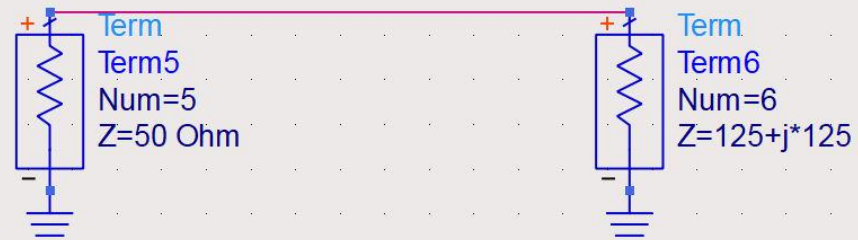
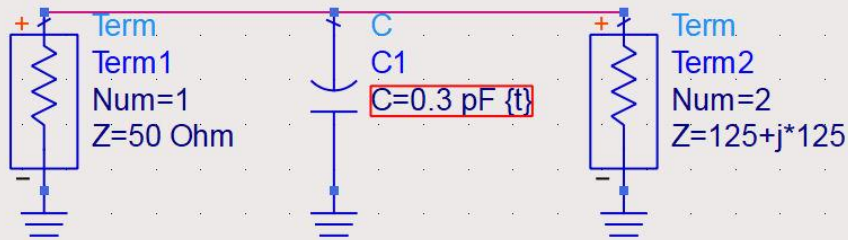
$$y_{in} = g_L + j \cdot (b_L + b_1)$$


$$g_{in} = g_L$$

$$j \cdot b_1 = j \cdot \omega \cdot C_1 \cdot Z_0 > 0$$

$$j \cdot b_1 = -j \cdot Z_0 / \omega \cdot L_1 < 0$$

# ADS, shunt susceptance



 **S-PARAMETERS**

S\_Param  
SP1  
Freq=1.0 GHz

Tune Parameters

Simulate  
While Slider Moves  
Tune

Parameters  
Include Opt Params  
Enable/Disable...  
 Display Full Name  
 Snap Slider to Step

Traces and Values  
Store... Recall...  
Trace Visibility...  
Reset Values  
Close Unassociated Data Displays

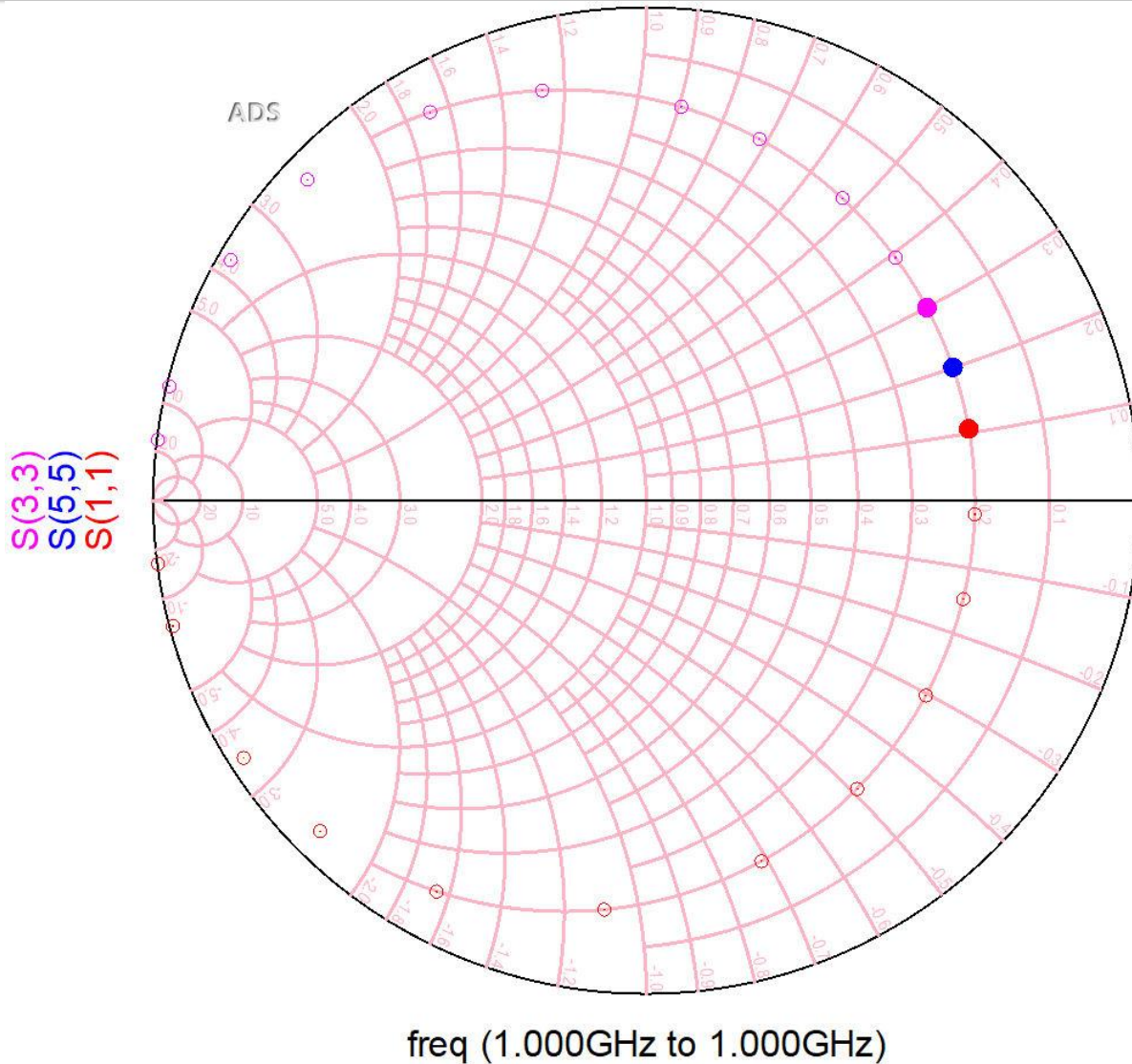
Update Schematic  
Close Help

adaptare\_IC\_lib:X:schematic

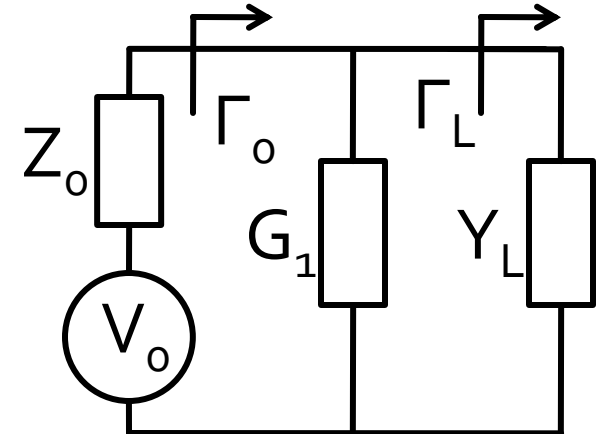
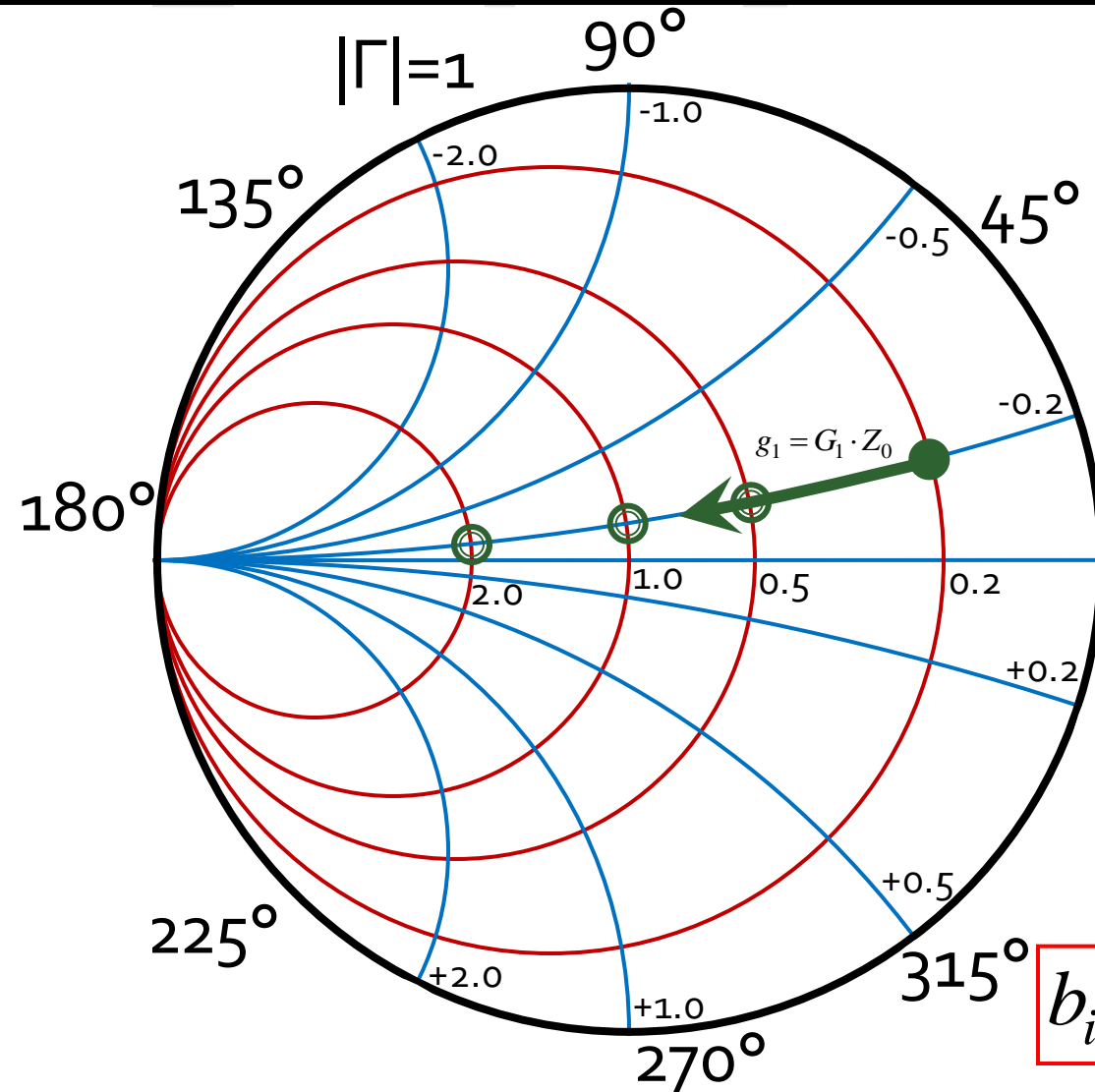
L1.L (nH)  
Value: 80  
Max: 100  
Min: 0.5  
Step: 0.1  
Scale: Lin

C1.C (pF)  
Value: 0.3  
Max: 50  
Min: 0.1  
Step: 0.1  
Scale: Lin

# ADS, shunt susceptance



# The Smith Chart, shunt conductance



$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678 \angle 23.5^\circ$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

$$Y_{in} = Y_L + G_1 = (G_L + G_1) + j \cdot B_L$$

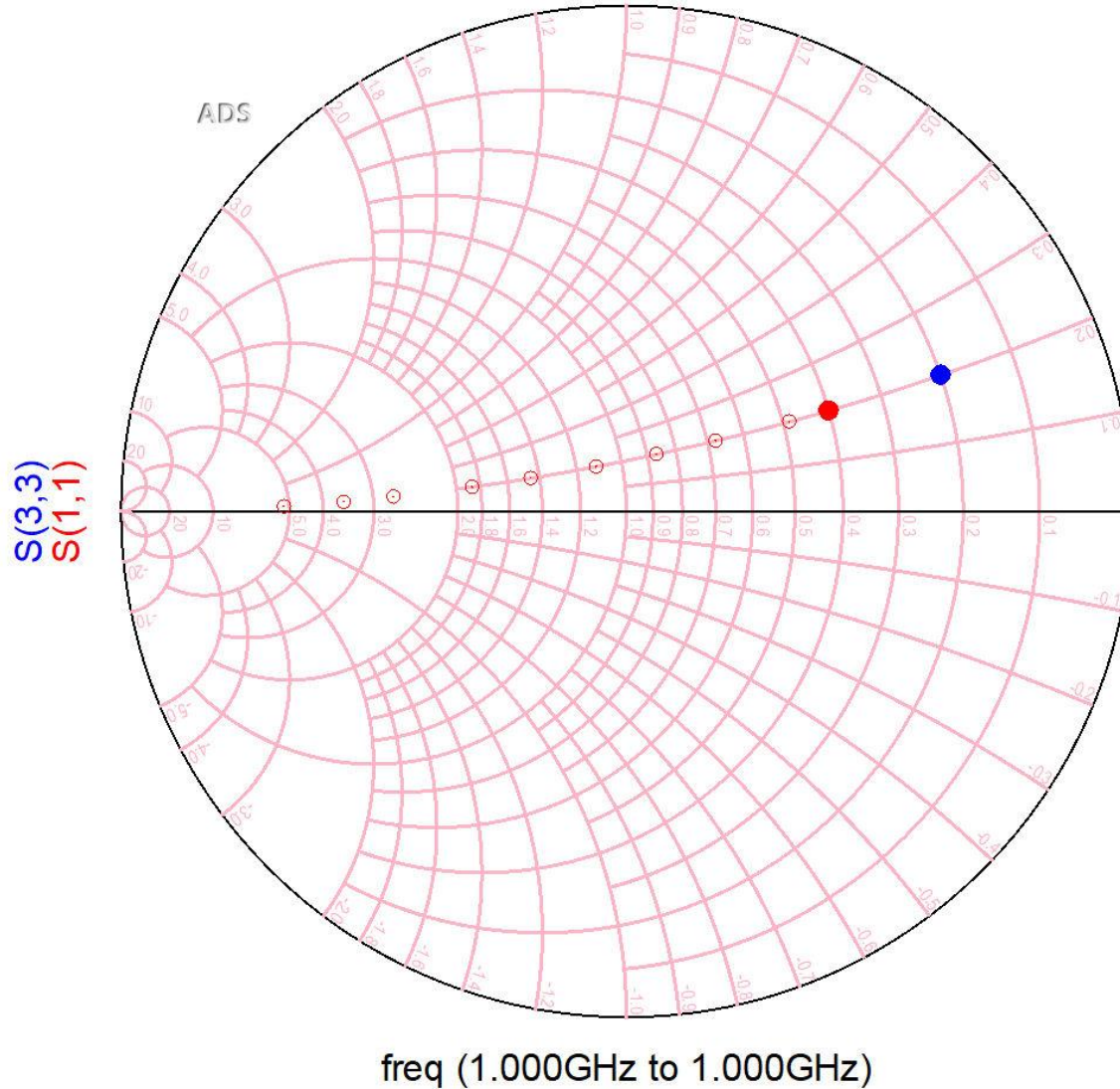
$$y_{in} = (g_L + g_1) + j \cdot b_L$$

$$b_{in} = b_L$$

$$g_{in} = g_L + G_1 \cdot Z_0$$



# ADS, shunt conductance



# Contact

- Microwave and Optoelectronics Laboratory
- <http://rf-opto.etti.tuiasi.ro>
- [rdamian@etti.tuiasi.ro](mailto:rdamian@etti.tuiasi.ro)